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Circuit for Balancing Harmonic-Polluted Three-Phase Networks

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***Summary*: This paper presents a new circuit capable of balancing harmonics-polluted three-phase power networks supplying single-phase loads. It takes into account the negative and positive sequences exhibited by the fifth and seventh harmonics, respectively. The concept extends the relations of the Steinmetz circuit, which are based exclusively on fundamental frequency considerations. The Steinmetz circuit does not, therefore, guarantee the balancing of the different harmonics in the line currents. Furthermore, it can lead to resonances between the capacitive and inductive elements in the two added balancing branches. A procedure for identifying the topology and the circuit parameters of the proposed connection is given. The results of a detailed case study are presented in order to demonstrate the superiority of the proposed circuit over the conventional Steinmetz connection.**

Key words:
harmonic analysis, power quality, load balancing, frequency scan, Steinmetz circuit

1. INTRODUCTION

Single-phase loads lead to the unbalanced operation of the power networks which are usually designed based on the ideal situation of balanced loading. Although a considerable percentage of the loads in these networks are balanced three-phase ones, the adverse effects of eventually existing single-phase loads were the topics of many investigations under both balanced [1–4], and unbalanced supply conditions [5]. For instance, the single-phase load of the pure real admittance (or conductance) $Y_{bc} = G$ Siemens (or Ω^{-1}) across the terminals *b* and *c* of the three-phase supply shown in Fig. 1 will lead to the extremely unbalanced currents: $I_a = 0$, $I_b = GV$, and $I_c = -GV$, where V is the network's rated line voltage, with the voltage V_{bc} as reference.

Adding the capacitive and inductive susceptances $Y_{ca} = +jG/\sqrt{3}$ and $Y_{ab} = -jG/\sqrt{3}$ across the line terminals *c,a* and *a,b,* respectively, as depicted in Fig. 2, will result in the Steinmetz circuit. It can be shown that the new line currents I_a, I_b and I_c will then constitute a positive sequence system of three-phase currents having equal magnitudes of $GV/\sqrt{3}$ and 120 degrees phase shifts, [1].

Although most of the studies in the literature address the operation of the Steinmetz balancing circuit under pure sinusoidal operating conditions [1–5], several recent investigations deal with the harmonic performance and the potential resonance problems in power networks comprising such balancing circuits [6–13]. For example, the theoretical and experimental studies in refer ences [8–10] address the high frequency response as well as the possible parallel and series resonances that can take place between the capacitor of the Steinmetz circuit and the power network's inductance. References [11,13] investigate the increase in the voltage distortion and the identification and mitigation of the possible harmonic disturbances resulting from the background voltage distortion. Relations were presented for identifying the series resonance appearing, if a Steinmetz circuit is connected to a harmonic-polluted supply and for cases involving nonlinear loads injecting harmonic currents.

It should be noted that the balancing capability of the Steinmetz circuit depends on two important conditions:

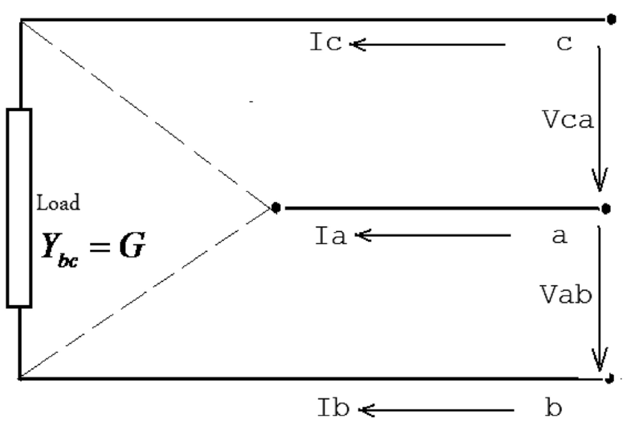

Fig. 1. A single–phase load of conductance *G* connected to the terminals *b, c* of a symmetrical three-phase network of line voltage *V.* [1]

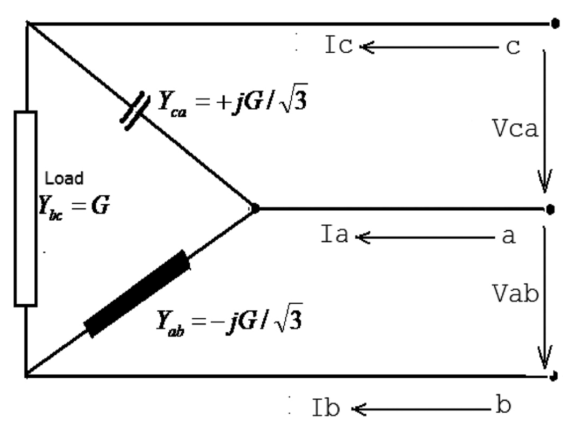

Fig. 2. The Steinmetz circuit comprising the two balancing reactive elements $Y_{ca} = +jG/\sqrt{3}$ and $Y_{ab} = -jG/\sqrt{3}$ used for balancing the single–phase load of conductance *G* connected to the terminals *b, c* of a symmetrical three-phase network of line voltage *V.* [1]

(i) The phase sequence of the supply voltage should be positive (i.e. *a-b-c*).

(ii) The supply frequency should be equal to that used for the calculation of the capacitance C forming Y_{ca} connected between *c&a* and the inductance L forming Y_{ab} connected between *a&b*. In other words, $G/\sqrt{3}=\omega_o C$ and $G/\sqrt{3}=1/(\omega_o L)$ where ω_0 is the fundamental angular frequency.

With regard to the harmonic response of the balancing circuit, it should be taken into consideration that the harmonics of theorders $h=6p+1$ ($p=1,2,3,$etc.), i.e. $h=7,13,19,...,$are of a positive sequence, whereas those having $h=6p-1$ ($p=1,2,3,$etc.), i.e. $h=5,11,17,....,$ exhibit the characteristics of a negative sequence system.

Fig. 3 shows the waveforms of the line currents I_a, I_b and I_c for the single phase resistive loading described by the circuit depicted in Fig. 1. Currents are given in per unit based on the *rms* load current (VG). The 60-Hz supply voltage is assumed to include fifth and seventh harmonic components of 4% and 3%, respectively. As expected, $I_a=0$ (thinnest curve), while the two other line currents I_b (curve of medium thickness) and I_c (thickest curve) are identical but of opposite signs.

Because the load is purely resistive, and due to the absence of any inductances or capacitances in the unbalanced circuit of Fig.1, the line currents in Fig. 3 exhibit the same percentage harmonic content and distortion factor of the supply voltage. This should be compared with the three strongly distorted line currents resulting from the use of the Steinmetz balancing circuit given in Fig. 4. The fundamental components of I_a, I_b and I_c constitute a three-phase balanced system, while the much higher harmonic content of any of the three line currents is due to the fact that the two above mentioned conditions i) and ii) are not satisfied for the fifth and seventh harmonics.

Fig. 5 illustrates the dependence of the amplitudes of the harmonic currents in the three line currents I_a, I_b and I_c on the harmonic order (or the per unit frequency) h. Only integer odd harmonic orders h are relevant in this plot. As expected, the fundamental components of the three line currents are equal to $1/\sqrt{3}$ per unit based on GV. Regarding the fifth harmonics, it can be seen that their percentage in the line currents I_a, I_b and I_c are, respectively, 2.8, 1.2 and 3.8 times higher than those in the supply voltage. The corresponding magnification factors for the seventh harmonic are 4.1, 0.9 and 3.5, respectively.

The fifth and seventh harmonic components in the supply voltage constitute symmetrical 3-phase systems of negative and positive sequences, respectively. The per unit magnitudes of the negative sequence components in the line currents are given in Fig. 6 versus the harmonic order. Of special interest are the values corresponding to $h=1$ (fundamental), $h=5$ and $h=7$. The plot shows the absence of any negative sequence component for $h=1$, since the Steinmetz circuit was originally designed for the fundamental frequency. The corresponding values for $h=5$ and $h=7$ are 2.227 and 1.874 per unit, respectively.

2. THE PROPOSED CIRCUIT

The above comments indicate that the adverse harmonic effects of the Steinmetz connection are due to the fact that the admittances of the two added branches $Y_{ca}=+jG/\sqrt{3}$ and $Y_{ab}=-jG/\sqrt{3}$, shown in Fig. 2, were determined with the objective of balancing the circuit at the fundamental frequency ω_0. In the following section a modified balancing circuit is suggested for balancing the circuit at the power frequency ω_0 and, simultaneously, capable of mitigating the adverse harmonic effects associated with the first two characteristic harmonics (i.e. the fifth and the seventh). The new balancing admittances $Y_{ca,new}$, $Y_{ab,new}$ will now be selected such that the circuit will be balanced at the fundamental frequency ω_0, the fifth as well as the seventh harmonic frequencies 5 ω_0 and 7 ω_0, respectively. Accordingly, the six following conditions must be satisfied by $Y_{ca,new}$, $Y_{ab,new}$.

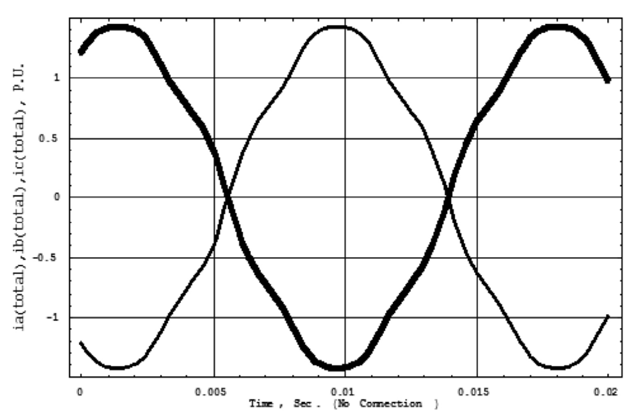

Fig. 3. Plots of the three line currents I_a, I_b and I_c for the unbalanced condition shown in Fig. 1 with a supply voltage polluted by 4% fifth and 3% seventh harmonics, respectively. Currents are in per unit based on the *rms* load current (VG)
$I_a=0$ (thinnest curve), I_b (curve of medium thickness) and I_c (thickest curve)

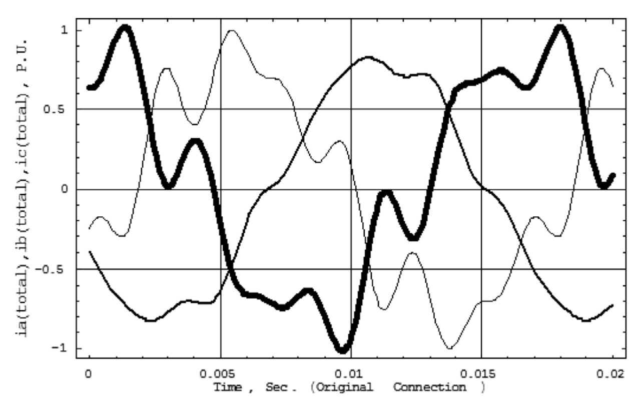

Fig. 4. Plots of the three line currents I_a, I_b and I_c for the balanced condition shown in Fig. 2 (using the Steinmetz circuit) with a supply voltage polluted by 4% fifth and 3% seventh harmonics, respectively. Currents are in per unit based on VG
$I_a=0$ (thinnest curve), I_b (curve of medium thickness) and I_c (thickest curve)

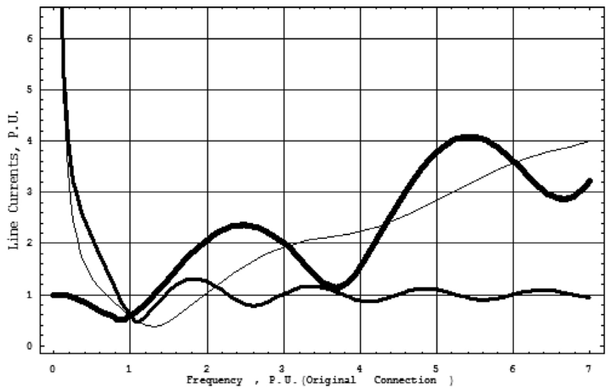

Fig. 5. Plots of the frequency characteristics of the three line currents I_a, I_b and I_c for the balanced condition shown in Fig. 2 (using the Steinmetz circuit).
I_a (thinnest curve), I_b (curve of medium thickness) and I_c (thickest curve)

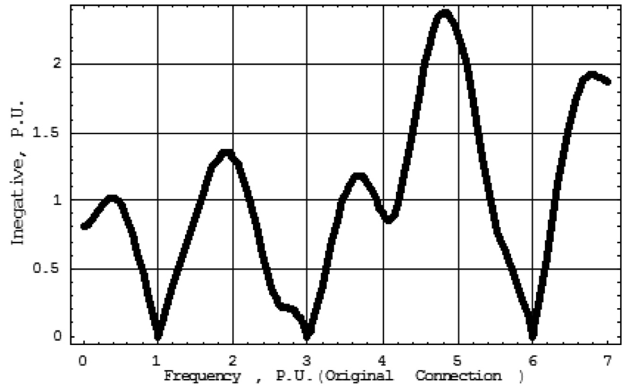

Fig. 6. Plots of the negative sequence component in the three line currents I_a, I_b and I_c for the circuit shown in Fig. 2 (i.e. using the Steinmetz circuit) versus the harmonic order (or the per unit frequency) *h*.

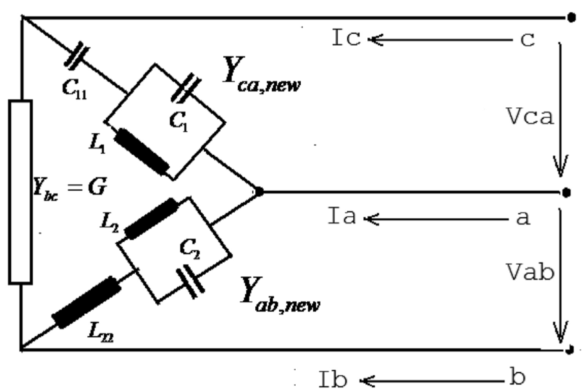

Fig. 7. The modified balancing circuit.

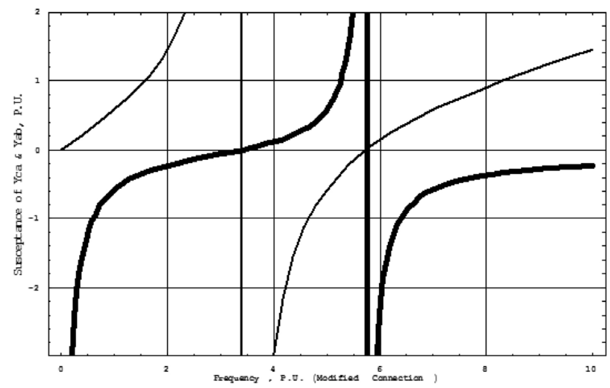

Fig. 8. The frequency response of the per unit susceptances of $Y_{ca,new}$, $Y_{ab,new}$.
thin curve: susceptance of $Y_{ca,new}$, thick curve: susceptance of $Y_{ab,new}$

$$
\begin{aligned}
Y_{ca,new}(\omega_o) &= +jG/\sqrt{3} \\
Y_{ab,new}(\omega_o) &= -jG/\sqrt{3} \\
Y_{ca,new}(5\omega_o) &= -jG/\sqrt{3} \\
Y_{ab,new}(5\omega_o) &= +jG/\sqrt{3} \\
Y_{ca,new}(7\omega_o) &= +jG/\sqrt{3} \\
Y_{ab,new}(7\omega_o) &= -jG/\sqrt{3}
\end{aligned}
\tag{1}
$$

The signs of the admittances take the positive sequence nature of both the fundamental and the seventh harmonic as well as the negative sequence of the fifth harmonic into account. The availability of the above six equations implies that a modified balanced circuit can be identified, including three elements within each of the new admittance branches, such as the suggested circuit shown in Fig. 7. It includes the still unknown 6 circuit elements (C_{11}, L_1, C_1), and (L_{22}, L_2, C_2) constituting $Y_{ca,new}$, $Y_{ab,new}$, respectively.

At any angular frequency, ω, the two new admittances are given by:

$$Y_{ca,new}(\omega) = [1/(j\omega C_{11}) + (j\omega L_1)/((j\omega)^2 L_1 C_1 + 1)]^{-1} \tag{2}$$

$$Y_{ab,new}(\omega) = [(j\omega L_{22}) + (j\omega L_2)/((j\omega)^2 L_2 C_2 + 1)]^{-1} \tag{3}$$

After substituting these two relations in the previously listed six simultaneous equations (1), an analytical closed-form solution could be found using the software *Mathematica* as:

$$C_{11} = 0.544359(\frac{G}{\omega_o}) \tag{4}$$

$$C_1 = 0.297696(\frac{G}{\omega_o}) \tag{5}$$

$$L_1 = \frac{0.101792}{G\omega_o} \tag{6}$$

$$L_{22} = \frac{0.57735}{G\omega_o} \tag{7}$$

$$L_2 = \frac{1.05573}{G\omega_o} \tag{8}$$

$$C_2 = 0.0811899(\frac{G}{\omega_o}) \tag{9}$$

Where ω_0 is the fundamental angular frequency in rad./s.

3. SAMPLE RESULTS

The effectiveness of the above suggested balancing circuit and the validity of the solution technique are demonstrated by Fig. 8. It gives two curves for the frequency response of the per unit susceptances (i.e. the imaginary parts of $Y_{ca,new}$, $Y_{ab,new}$). They are depicted by the thin and the thick curves, respectively. The base value of the susceptances is the conductance G of the single-phase load. The curves show that at the fundamental , the 5th harmonic and the 7th harmonic frequencies, the two admittances exhibit equal susceptances' magnitudes (0.57 per unit or ,equivalently, $G/\sqrt{3}$ Siemens) but of opposite signs, as expected.

The improvement achieved through the use of the suggested balancing circuit of Fig. 7 is manifested in the waveforms of the three line currents I_a, I_b and I_c depicted in Fig. 9. The equations of these line currents are given in the Appendix. Comparing these curves with those given earlier in Fig. 4 (obtained using the Steinmetz circuit), it can be clearly seen that these new waveforms are balanced and their shapes are much closer to the pure sine waves of the fundamental power frequency. In fact the new time curves of the line currents are similar to the waveform of any of the supply line voltages as well as to the line currents I_b and I_c in Fig. 3.

Fig. 10 illustrates the per unit amplitudes of the modified harmonic components in the three line currents I_a, I_b and I_c resulting from the application of the suggested balancing circuit of Fig. 7 versus the harmonic order h. Only integer odd harmonic orders h are relevant in this plot. The curves show that for each of the fundamental ($h=1$) as well as the 5th and 7th harmonic frequencies (i.e. $h=5$ and $h=7$), the per unit amplitudes of the harmonic components in the three line currents and are equal. Results of computing the phase angles indicated also that the phase shifts between each two of the line currents are $2\pi/3$ radians in the corresponding phase sequence (i.e. negative for $h=5$ and positive for $h=7$), This implies that the suggested circuit is capable of balancing the 5th and 7th harmonic currents in addition to those of the power frequency. The superiority of the modified balancing circuit of Fig. 7 can be recognized by comparing the plots of Fig.10 with those depicted in Fig. 5 for the Steinmetz circuit

The per unit magnitudes of the negative sequence components in the line currents, as functions of the harmonic order h, are given in Fig. 11. Of special interest are the values corresponding to $h=1$ (fundamental), $h=5$ and $h=7$. The plot shows zero values of the negative sequence components for all of them, since the new balancing admittances $Y_{ca,new}$, $Y_{ab,new}$ were selected, according to Eq. (1), such that the circuit will be balanced at these three frequencies. The advantage of the suggested circuit will be recognized if this plot is compared with that shown in Fig.6 for the Steinmetz connection.

It is seen from Eqs. (1) That there are two new conditions which can be formulated for each additionally considered harmonic. With these new equations it will be possible to identify two new circuit elements. They must be reactive (in order to avoid ohmic losses), and they should be of opposite nature (i.e. an inductance in one of the two branches *ab* or *ca* and a capacitance in the other branch).

The extension of the proposed balancing procedure in order to take one more of the background harmonics of the supply voltage ($h=11$ for example) into account is based on the addition of one more reactive element to each of the two admittances $Y_{ca,new}$, $Y_{ab,new}$. Accordingly, two more equations should be added to Eqs. (1), and solved simultaneously. They are: $Y_{ca,new}(11\omega_o) = -jG/\sqrt{3}$, $Y_{ab,new}(11\omega_o) = +jG/\sqrt{3}$.

It is believed that it is possible to design different *LC* structures.

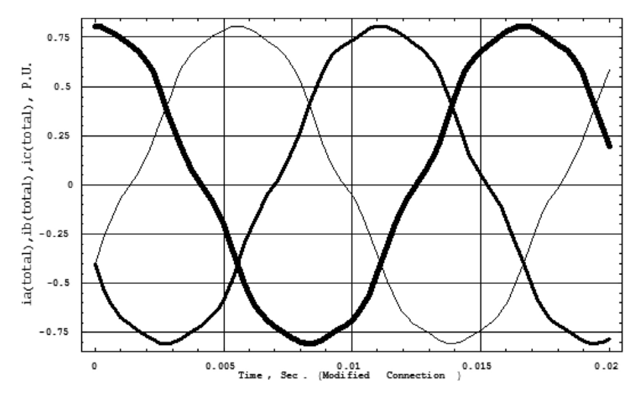

Fig. 9. Plots of the three line currents I_a, I_b and I_c using the suggested circuit given in Fig. 7 with a supply voltage polluted by 4% fifth and 3% seventh harmonics, respectively. Currents are in per unit based on *VG*
I_a (thinnest curve), I_b (curve of medium thickness) and I_c (thickest curve)

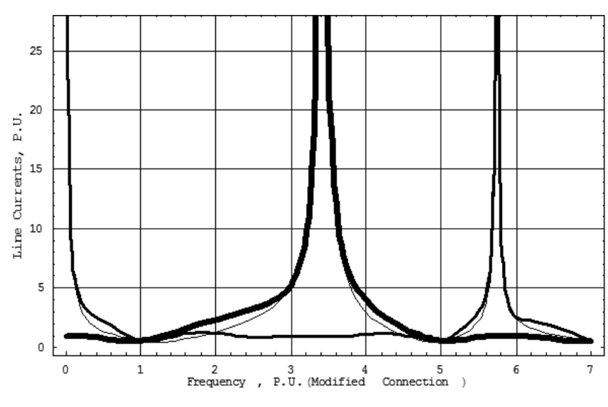

Fig. 10. Plots of the per unit harmonic content in the three line currents I_a, I_b and I_c for the balanced condition shown in Fig.2 (using the modified circuit shown in Fig. 7).
I_a (thinnest curve), I_b (curve of medium thickness) and I_c (thickest curve)

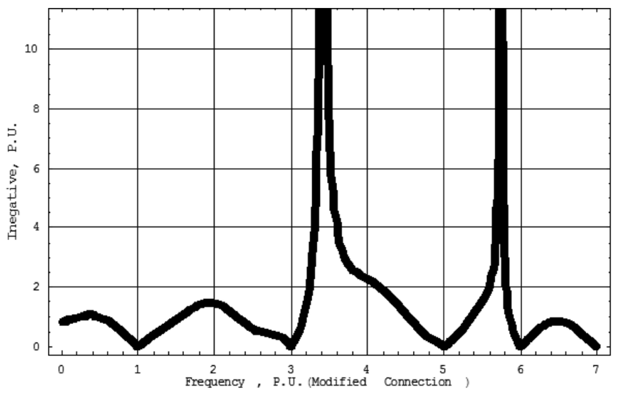

Fig. 11. Plots of the per unit negative sequence component in the line currents I_a, I_b and I_c for the circuit shown in Fig. 7 (i.e. using the modified circuit) versus the per unit frequency.

4. CONCLUSIONS

- The Steinmetz circuit does not guarantee the balancing of the different harmonics in the line currents. For a single-phase resistive load connected across the two lines b and c, three strongly distorted line currents will result. Regarding the fifth harmonics, their percentage in the line currents I_a, I_b and I_c are, respectively, 2.8, 1.2 and 3.8 times higher than those in the supply voltage. The corresponding magnification factors of the seventh harmonic are 4.1, 0.9 and 3.5, respectively. Results indicate the absence of any negative sequence component for $h=1$, since the Steinmetz circuit is designed for the fundamental frequency. The corresponding values for $h=5$ and $h=7$ are 2.227 and 1.874 per unit, respectively.
- A modified circuit is suggested for balancing the power network at the power frequency ω_0 and is, simultaneously, capable of mitigating the adverse effects resulting from the fifth and the seventh harmonics in the supply voltage. The new balancing admittances will now be selected such that the network will be balanced at $h=1$, $h=5$ and $h=7$.
- The results of a case study are presented in order to validate the performance of the proposed circuit and to demonstrate its superiority over the conventional Steinmetz connection. The improvement is reflected in the waveforms of the three line currents. They are more balanced and much closer to the pure sine waves of the fundamental power frequency. The new time curves of the line currents are similar to the wave form of the supply line voltages.
- For each of the fundamental as well as the 5th and 7th harmonic frequencies, the amplitudes of the harmonic components in the three line currents I_a, I_b and I_c are equal with $2\pi / 3$ radians phase differences. This implies that the suggested circuit is capable of balancing the 5th and 7th harmonic currents in addition to those of the power frequency.
- The dependence of the magnitudes of the negative sequence components in the line currents on the harmonic order h was investigated. Of special interest are the values corresponding to $h=1$ (fundamental), $h=5$ and $h=7$. Results show a zero value of the negative sequence for all of them.

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APPENDIX

Relations for the Line Currents:

Considering the circuit in Fig. 2, the following equations can be obtained using Ohm's and Kirchhoff's laws:

$$I_a = V_{ab}Y_{ab} - V_{ca}Y_{ca} \tag{A-1}$$

$$I_b = V_{bc}G - V_{ab}Y_{ab} \tag{A-2}$$

$$I_c = V_{ca}Y_{ca} - V_{bc}G \tag{A-3}$$

For the modified balancing circuit in Fig. 7, the corresponding relations will be:

$$I_a = V_{ab}Y_{ab,new} - V_{ca}Y_{ca,new} \tag{A-4}$$

$$I_b = V_{bc}G - V_{ab}Y_{ab,new} \tag{A-5}$$

$$I_c = V_{ca}Y_{ca,new} - V_{bc}G \tag{A-6}$$

It should be noted that once the modified circuit in Fig. 7 is analyzed, the results corresponding to the Steinmetz circuit in Fig. 2 can be easily obtained by substituting the following special admittance values:

$$C_{11} = G / (\omega\sqrt{3}) \quad L_1 = 0 \qquad L_{22} = \sqrt{3} / (\omega G) \quad \text{and} \quad L_2 = 0$$

The original unbalanced circuit involving only the single phase load (Fig. 1) can be simulated by inputting the following special values: $C_{11} = 0$ and $L_{22} = \infty$.

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