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On Some Properties of Instantaneous Active and Reactive Powers

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***Summary*: Some features of the instantaneous active and reactive powers p and q , not reported earlier, are discussed in this paper. It is shown that oscillating components $\tilde{p}$ and $\tilde{q}$ of these powers may not depend on the order of the load current harmonic. It was shown, moreover, that a load supplied with a nonsinusoidal voltage may not be distinguished in terms of p and q powers from a harmonics generating load (HGL) supplied with a sinusoidal voltage. This observation is important for compensator control. Also it was shown that the instantaneous reactive power q cannot be interpreted as a measure of amount of energy exchanged between supply lines of a load or as amount of energy rotating around such lines, as it was suggested in some publications on the IRP p-q Theory. A phenomenon of energy rotation or exchange between lines does not exist.**

Key words:
Active power filters,
CPC theory,
harmonics,
nonsinusoidal systems,
switching compensators

I. INTRODUCTION

The Instantaneous Reactive Power (IRP) p-q Theory, developed by Nabae, Akagi and Kanazawa in Ref. [1], is one of the main power theories of three-phase systems at nonsinusoidal conditions. This theory, regarded as an algorithm of switching compensators control, was compared in Ref. [2] to the Currents' Physical Components (CPC) power theory, developed by the author of this paper [11]. Paper [2] was written from the perspective of the p-q theory and does not report some properties of p and q powers important for practical applications at compensation, however. Some such properties, not reported earlier, will be discussed in this paper.

One of the major objectives of power theories and the main motivation for their development is explanation of how a nonsinusoidal supply voltage and the load originated current harmonics affect power properties of such systems and description them in terms of powers. Such description can be next used for design and control of compensators capable of improving power properties of electrical systems.

The sources of the voltage distortion in first decades of power systems development were not very common and relatively of low power, so that distribution voltage was not substantially distorted. Therefore, studies on effects of this distortion on power properties of electrical systems had a cognitive significance rather than a practical one.

First answers were provided by Budeanu [3], Fryze [4], Shepherd [6], or Kusters & Moore [8]. Studies were extended by Quade [5], Depenbrock [7], Czarnecki [11] and others, to three-phase systems. One of the major power theories of three-phase systems is the IRP p-q Theory.

Development of power electronics, retrofitting incandescent bulbs with fluorescent ones, omnipresence of rectifiers in video, computers or microwave stoves, meaning, *harmonic generating loads* (HGLs), changes this situation. These devices are sources of current harmonics on the customer side and consequently, harmonics in distortion voltage. Voltage harmonics can affect power properties of electrical loads, thus development of power theories has now not only cognitive, but also practical significance. They should describe power properties of electrical systems in the presence of the supply voltage harmonics. The plural form "*theories*" is used here, because there are a number of substantially different approaches [18] to explanation and description of power properties of electrical systems, thus different power theories.

Traditionally, current harmonics generated by high power industrial loads, such as rectifiers, are reduced by resonant harmonic filters (RHFs). In the presence of distribution voltage harmonics, such filters are sometimes not capable [14], however, of reducing the voltage and current distortion and have to be replaced by switching compensators (SCs), also known as "active power filters", Thus, SCs, which supersede RHFs, should operate correctly at the voltage distortion which makes RHFs useless for harmonics reduction.

II. INSTANTANEOUS p AND q POWERS

Power properties of three-phase loads supplied from three-wire lines are specified in the IRP p-q Theory in terms of only two powers: the instantaneous active and reactive p and q powers. They are defined in terms of the load voltages and currents in α and β coordinates, calculated with the Clarke's Transform.

Some properties of the IRP p-q Theory, important from the cognitive perspective, were discussed in Ref. [15]. Only one cognitive issue, related to hypothetical energy rotation, is discussed in this paper, however. Discussion on possible other physical interpretations of the q-power is beyond the scope of this paper. Some properties, not previously reported, of the oscillating components of the p and q–powers, that might affect results of the SCs control, are the main subject of this paper.

The Clake's Transform for three-phase, three-wire systems with line-to-artificial zero voltages, meaning such that

$$u_{\mathrm{R}} + u_{\mathrm{S}} + u_{\mathrm{T}} \equiv 0,$$

has the form:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix}, \tag{1}$$

and similarly, for the line currents, since $i_R + i_S + i_T \equiv 0$,

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix}. \tag{2}$$

The instantaneous active power in such coordinates, is defined as

$$p = u_\alpha i_\alpha + u_\beta i_\beta, \tag{3}$$

while the instantaneous reactive power, q, is defined in as

$$q = u_\alpha i_\beta - u_\beta i_\alpha. \tag{4}$$

The instantaneous powers can be calculated instantaneously, only with a small delay needed for a few arithmetical operations. However, when these powers are applied for a switching compensator control, then separation of their average and varying components:

$$p = \bar{p} + \tilde{p}, \tag{5}$$

$$q = \bar{q} + \tilde{q}, \tag{6}$$

is usually needed. The average values are defined over one period T of the supply voltage, thus one period is needed for decompositions (5) and (6), although there are situations where this time can be reduced.

The instantaneous active power p is nothing else than the instantaneous power, meaning the rate of energy $W(t)$ flow to electrical loads. For symbols shown in Fig. 1, it is defined as

$$p(t) = \frac{dW(t)}{dt} = u_R(t)\, i_R(t) + u_S(t)\, i_S(t) + u_T(t)\, i_T(t), \tag{7}$$

and was introduced to electrical engineering well before the IRP p-q Theory was developed. The change of the traditional name of this power to *instantaneous* ***active*** *power* is not fortunate, however, because electrical loads always have this power, independently whether they do have any active power P or not, as it is with purely reactive LC loads.

III. HYPOTHETICAL ENERGY ROTATION OR EXCHANGE

To have a cognitive value, a power theory should provide a physical interpretation of quantities used by that theory for describing power properties.

The physical meaning of the instantaneous reactive power q, in early papers on IRP p-q Theory was not provided. The only explanation of the physical meaning of the q-power, presented by Akagi and Nabae in Ref. [9] has the form:

"*The instantaneous imaginary power q was introduced on the same basis as the conventional instantaneous real power p in three-phase circuits, and then the instantaneous reactive power was defined with the focus on the physical meaning and the reason for naming.*"

This sentence does not explain, of course, the physical meaning of the IRP q. Thus, in spite of the verbal declaration referenced above, physical interpretation of instantaneous reactive power was not provided.

The following interpretation of the instantaneous reactive power q was provided recently in the book: *Instantaneous Power Theory and Applications to Power Conditioning* [12] by Akagi, Watanabe and Aredes:

"*…the imaginary power q is proportional to the quantity of energy that is being exchanged between the phases of the system…" "Figure".." summarizes the above explanations about the real and imaginary powers.*"

This Figure with original caption, copied from that book, is shown in Fig. 2.

The Reader may observe, however, that presented explanation of the meaning of the imaginary power q does not fit Fig. 2, because in the text is told: "*energy is being exchanged between phases*" while Fig. 2 is drawn in such a way as if this energy rotates around the supply line. Because this is not clear, we should verify if any of these flows of energy is possible.

A picture with energy rotating around supply lines, marked with q, repeats in number of figures in book [12] and even on the book cover, sending a strong message to electrical engineering community that the IRP q occurs due to energy rotation around lines or due to a sort of energy exchange between them. This message is erroneous, however.

Flow of energy in electromagnetic fields was described [13] by J.H. Poynting in 1884. He introduced the concept

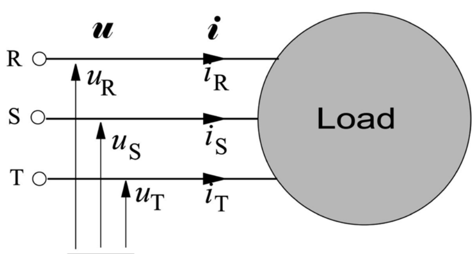

Fig. 1. Three-phase load supplied with a three-wire line

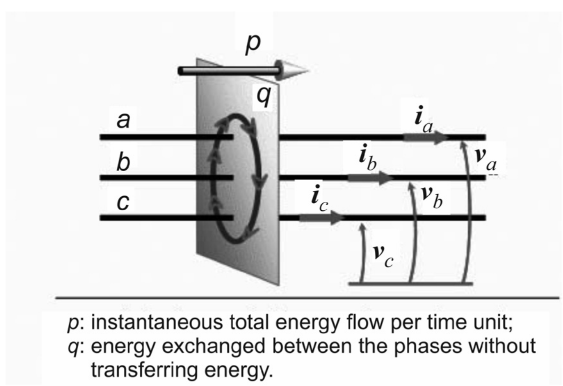

Fig. 2. Physical meaning of the instantaneous active and reactive powers.

of the Poynting Vector, specified as a vector product of the electric and magnetic field intensities, $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$, namely

$$\vec{\boldsymbol{P}} = \vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}. \tag{8}$$

This vector specifies direction of the surface density of the rate of energy flow in electromagnetic fields. Namely, the rate of energy flow through surface of area A is equal to

$$\iint_{\boldsymbol{A}} \vec{\boldsymbol{P}} \bullet d\vec{\boldsymbol{A}} = \frac{dW(t)}{dt}, \tag{9}$$

and this energy flows in the direction of the Poynting Vector, which is at any point of a space perpendicular to the plane spanned on the vectors of electric and magnetic fields intensities at that point. It is perpendicular to each of them, as shown in Fig. 3.

Now, let us check whether the situation shown in Fig. 2 is possible or not, meaning does the energy rotate around the supply line?

Let us assume for that purpose, to simplify analysis, that the supply line, composed of conductors a, b and c, is a flat line. Magnetic field intensity at point x of lines plane is perpendicular to that plane as shown in Fig. 4.

If the energy rotates around such a line, then the Pointing Vector at that point should be perpendicular to that plane as well. The question sign in Fig. 4 emphasizes the question: "*is such a situation possible?*" It is not possible because the Poynting Vector cannot be parallel to the vector of the magnetic field intensity, however. Thus, the energy cannot rotate, as suggested in Fig. 2, around supply lines.

Let us verify whether "*energy is being exchanged between phases…*" as it was written in Ref. [12] or not. When conductors are ideal, meaning their resistance can be neglected, the electric field intensity is perpendicular to the conductor surface as shown in Fig. 5.

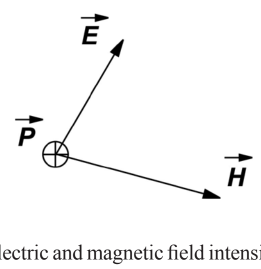

Fig. 3. Orientation of electric and magnetic field intensities and the Poynting Vector

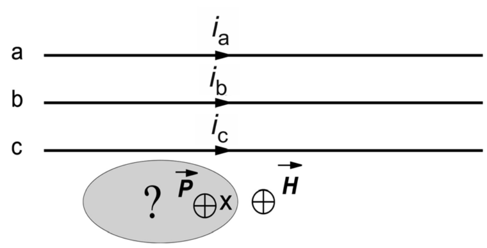

Fig. 4. Orientation of magnetic field intensity at point x in conductors plane

If the energy flows between phases, then the Poynting Vector should be perpendicular to conductors as well, meaning it would be parallel to the electric field intensity, which is not possible. It has to be perpendicular to this intensity. Consequently, the energy cannot flow, or be "exchanged" between phases. Only when a resistance of line conductors is not neglected, then the electric field intensity has a component along conductor surface and the Poynting Vector has a component towards conductors. In such a case, some amount of energy flows to conductors, where it is dissipated as heat. This dissipation has nothing in common with "*energy exchange between phases*", however.

Thus, this reasoning, based on a very fundamental principle of electromagnetic fields, demonstrates that the physical interpretation of the instantaneous reactive power *q*, suggested in Ref. [12], is not acceptable. There is no such physical phenomenon as the energy exchange between supply lines of three-phase loads. Similarly, energy does not rotate around them. In Fig. 2, there is such explanation: '*q*: *energy exchanged between the phases without transferring energy*'. It is difficult to guess, however, what the authors had in mind, since there is no explanation in Ref. [12] how energy could be *exchanged without its transfer*. It is also difficult to accept explanation of the meaning of the instantaneous power in the same figure, namely, '*p*: *instantaneous total energy flow per time unit'*. Taking into account that in a time unit, meaning, one second, there is approximately 50 periods of the voltage variation, '*total energy flow per time unit*' specifies the active power *P*, which is a constant number, rather than the instantaneous power $p(t)$, which is defined as the instantaneous rate of energy flow from the supply to the load, $p(t) = \mathrm{d}W/\mathrm{d}t$.

Thus, with the lack of physical interpretation of the instantaneous reactive power *q*, the IRP p-q Theory has a major cognitive deficiency. Unfortunately, it also has a major deficiency when used as a control algorithm of switching compensators in systems with nonsinusoidal voltage. This is demonstrated in the following section.

IV. DIRECT CALCULATION OF INSTANTANEOUS POWERS

According to the IRP p-q Theory the instantaneous active and reactive powers *p* and *q* are calculated in terms of the load voltages and currents expressed, using the Clarke's Transform, in α and β. This Transform is not needed for calculation instantaneous powers *p* and *q*, however. Voltages and currents in α and β coordinates are mathematical rather than physical entities, as they are in electrical systems. It is

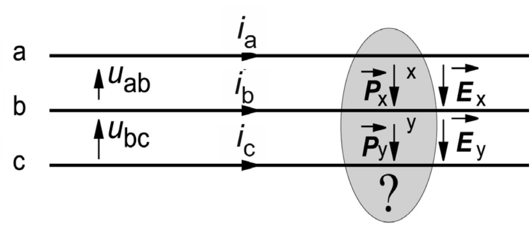

Fig. 5. Orientation of electric field intensity between conductors of three-phase line

easier to associate these powers with some features of the system when they are expressed directly, in terms of three-phase voltages and currents, rather than in terms of their values in α and β coordinates.

Formulas for direct calculation of the instantaneous active and reactive powers p and q are presented below. They can be simplified and made more compact using a concept of three-phase vectors of voltages and currents as introduced in Ref. [11], namely, a vector of supply voltages

$$\boldsymbol{u}(t) = [\,u_{\mathrm{R}}(t),\, u_{\mathrm{S}}(t),\, u_{\mathrm{T}}(t)]^{\mathrm{T}}, \tag{10}$$

where voltages at the load terminals R, S and T are measured with respect to an artificial zero, and similarly, a vector of supply currents

$$\boldsymbol{i}(t) = [\,i_{\mathrm{R}}(t),\, i_{\mathrm{S}}(t),\, i_{\mathrm{T}}(t)]^{\mathrm{T}}. \tag{11}$$

Since in three-phase, three-wire systems

$$i_{\mathrm{R}}(t) + i_{\mathrm{S}}(t) + i_{\mathrm{T}}(t) \equiv 0, \tag{12}$$

$$u_{\mathrm{R}}(t) + u_{\mathrm{S}}(t) + u_{\mathrm{T}}(t) \equiv 0, \tag{13}$$

the formula for calculation of the instantaneous power p can be simplified as follows:

$$p = \overline{p} + \tilde{p} = \frac{dW}{dt} = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = u_{\mathrm{R}}\, i_{\mathrm{R}} + u_{\mathrm{S}}\, i_{\mathrm{S}} + u_{\mathrm{T}}\, i_{\mathrm{T}} =$$
$$(u_{\mathrm{R}} - u_{\mathrm{T}})\, i_{\mathrm{R}} + (u_{\mathrm{S}} - u_{\mathrm{T}})\, i_{\mathrm{S}} = u_{\mathrm{RT}}\, i_{\mathrm{R}} + u_{\mathrm{ST}}\, i_{\mathrm{S}}. \tag{14}$$

Also the instantaneous reactive power q can be expressed directly in terms of the supply voltages and currents, meaning in phase coordinates. Namely, from Eqs. (1) and (2) we obtain:

$$q = u_{\alpha}\, i_{\beta} - u_{\beta}\, i_{\alpha} =$$
$$= \sqrt{\frac{3}{2}}\, u_{\mathrm{R}}\, \Big(\frac{1}{\sqrt{2}} i_{\mathrm{R}} + \sqrt{2}\, i_{\mathrm{S}}\Big) - \Big(\frac{1}{\sqrt{2}} u_{\mathrm{R}} + \sqrt{2}\, u_{\mathrm{S}}\Big)\sqrt{\frac{3}{2}}\, i_{\mathrm{R}} =$$
$$= \sqrt{3}\,(u_{\mathrm{R}}\, i_{\mathrm{S}} - u_{\mathrm{S}}\, i_{\mathrm{R}}). \tag{15}$$

V. INSTANTANEOUS POWERS OF RESISTIVE BALANCED HGL AT SINUSOIDAL VOLTAGE

Supply currents at terminals of a harmonic generating load (HGL) are nonsinusoidal. They can have harmonics of order n from a set N. A three-phase vector of such currents can be expressed in the form. .

$$\boldsymbol{i}(t) = \sum_{n\in N} \boldsymbol{i}_n(t). \tag{16}$$

Due to the source internal impedance, such currents cause the load voltage distortion. At sufficiently strong supply source, this distortion can be neglected, however. It is assumed in this Section that the load voltages are sinusoidal, meaning composed of only fundamental harmonic, $\boldsymbol{u}_1(t)$. It is assumed, moreover, that these voltages are symmetrical and of the positive sequence.

At such assumption the formula for the instantaneous power can be expressed as follows

$$p = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}_1^{\mathrm{T}} \sum_{n\in N} \boldsymbol{i}_n = \sum_{n\in N} \boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{i}_n = \sum_{n\in N} p_n, \tag{17}$$

where

$$p_n = u_{\mathrm{RT1}}\, i_{\mathrm{R}n} + u_{\mathrm{ST1}}\, i_{\mathrm{S}n}. \tag{18}$$

Also the instantaneous reactive power q can be expressed as the sum of this power for individual harmonics, namely

$$q = u_{\alpha}\, i_{\beta} - u_{\beta}\, i_{\alpha} = \sqrt{3}(u_{\mathrm{R}} i_{\mathrm{S}} - u_{\mathrm{S}} i_{\mathrm{R}}) =$$
$$= \sqrt{3}(u_{\mathrm{R1}} \sum_{n\in N} i_{\mathrm{S}n} - u_{\mathrm{S1}} \sum_{n\in N} i_{\mathrm{R}n}) =$$
$$= \sqrt{3}(\sum_{n\in N} u_{\mathrm{R1}} i_{\mathrm{S}n} - \sum_{n\in N} u_{\mathrm{S1}} i_{\mathrm{R}n}) = \sum_{n\in N} q_n, \tag{19}$$

where

$$q_n = \sqrt{3}\,(u_{\mathrm{R1}} i_{\mathrm{S}n} - u_{\mathrm{S1}} i_{\mathrm{R}n}). \tag{20}$$

Let us calculate instantaneous powers of a resistive balanced load, shown in Fig. 6.

The load is supplied from a source of sinusoidal symmetrical voltage such that the line-to-ground voltage at terminal R is equal to

$$u_{\mathrm{R}} = u_{\mathrm{R1}} = \sqrt{2}\, U_1 \cos\omega_1 t, \tag{21}$$

generates 5$^{\mathrm{th}}$ order, symmetrical current harmonic, such that line R current is

$$i_{\mathrm{R}} = \sqrt{2}\, I_1 \cos\omega_1 t + \sqrt{2}\, I_5 \cos(5\omega_1 t + \alpha_5). \tag{22}$$

At such assumptions, line-to-line voltages have waveforms

$$u_{\mathrm{RT1}} = \sqrt{6}\, U_1 (\omega_1 t - 30^0), \tag{23}$$

$$u_{\mathrm{ST1}} = \sqrt{6}\, U_1 (\omega_1 t - 90^0). \tag{24}$$

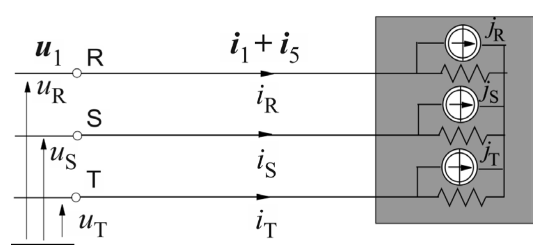

Fig. 6. Resistive balanced harmonic generating load (HGL) generating the 5th order current harmonic

Since the 5th order current harmonic is the negative sequence harmonic, then

$$i_S = \sqrt{2}\,[I_1\cos(\omega_1 t - 120^0) + I_5\cos(5\omega_1 t + \alpha_5 + 120^0)]\,. \quad (25)$$

At such conditions the instantaneous power is equal to

$$\begin{aligned} p &= u_{RT}\, i_R + u_{ST}\, i_S = \\ &= 2\sqrt{3}\,U_1\cos(\omega_1 t - 30^0)[I_1\cos\omega_1 t + I_5\cos(5\omega_1 t + \alpha_5)] + \\ &\quad + 2\sqrt{3}\,U_1\cos(\omega_1 t - 90^0)[I_1\cos(\omega_1 t - 120^0) + \\ &\quad + I_5\cos(5\omega_1 t + \alpha_5 + 120^0)] = \\ &= 3U_1 I_1 + 3U_1 I_5\cos(6\omega_1 t + \alpha_5) = \\ &= p_1 + p_5 = \bar{p} + \tilde{p}\,. \end{aligned} \quad (26)$$

The calculated instantaneous power is decomposed directly into the constant and oscillating components. These components were obtained explicitly only because waveforms of voltages and currents as well as rms values U_1, I_1 and I_5 were assumed to be known. Without Fourier analysis these values explicitly are not known, however. When the IRP p-q Theory is used for a compensator control, these powers are calculated, according to formulas (3) and (4), by a digital signal processing (DSP) system. It has sequences of voltages and currents samples as the input data which are first used for calculating voltages and currents in α and β coordinates, according to $\bar{p}$ and $\tilde{p}$ (1) and (2). The constant and oscillating components, are not seen explicitly in the instantaneous power p calculated by the DSP system, however. A low-pass or a high-pass filter is needed for decomposition of this power into the constant and oscillating components. The result of filtering for the situation as discussed has the form:

$$\bar{p} = A\,, \qquad \tilde{p} = B\cos(6\omega_1 t + \alpha_5), \quad (27)$$

but it is not known what contributes to values A and B.

The instantaneous reactive power of such a load, according to eqn. (19) is

$$q = q_1 + q_5, \quad (28)$$

where

$$q_1 = 0,$$

because the load shown in Fig. 6 is balanced for the fundamental harmonic and is purely resistive, while

$$\begin{aligned} q_5 &= \sqrt{3}\,(u_{R1}i_{S5} - u_{S1}i_{R5}) = \\ &= 2\sqrt{3}U_1\{\cos\omega_1 t\,[I_5\cos(5\omega_1 t + \alpha_5 + 120^0)] - \\ &\quad - \cos(\omega_1 t - 120^0)[I_5\cos(5\omega_1 t + \alpha_5)]\} = \\ &= -\,3U_1 I_5\sin(6\omega_1 t + \alpha_5). \end{aligned} \quad (29)$$

To check how the order n of the load originated harmonic affects instantaneous powers, let us assume that all other parameters of the circuit in Fig. 6 are kept unchanged, but instead of the 5th order harmonic, the load shown in Fig. 7, generates the 7th order harmonic.

Since the 7th order current harmonic is of the positive sequence then the current in line S is

$$i_S = \sqrt{2}\,[I_1\cos(\omega_1 t - 120^0) + I_7\cos(7\omega_1 t + \alpha_7 - 120^0)]\,. \quad (30)$$

The instantaneous power of the fundamental harmonic, p_1, remains, of course, unchanged, while

$$\begin{aligned} p_7 &= u_{RT1}\, i_{R7} + u_{ST1}\, i_{S7} = \\ &= 2\sqrt{3}\,U_1 I_7\,\{\cos(\omega_1 t - 30^0)[\cos(7\omega_1 t + \alpha_7)] + \\ &\quad + \cos(\omega_1 t - 90^0)[\cos(7\omega_1 t + \alpha_7 - 120^0)]\} = \\ &= 3U_1 I_7\cos(6\omega_1 t + \alpha_7) = \tilde{p}\,. \end{aligned} \quad (31)$$

Thus, the change in the order of the load generated harmonic from $n = 5$ to $n = 7$ does not affect the instantaneous power. After filtering the DSP output we obtain as previously

$$\bar{p} = A\,, \qquad \tilde{p} = B\cos(6\omega_1 t + \alpha_7), \quad (32)$$

thus, only the angle α can be different.

The instantaneous reactive power associated with the presence of the 7th order load originated current harmonic is

$$\begin{aligned} q_7 &= \sqrt{3}(u_{R1}i_{S7} - u_{S1}i_{R7}) = \\ &= 2\sqrt{3}U_1\{\cos\omega_1 t\,[I_7\cos(7\omega_1 t + \alpha_7 - 120^0)] - \\ &\quad - \cos(\omega_1 t - 120^0)[I_7\cos(7\omega_1 t + \alpha_7)]\} = \\ &= 3U_1 I_7\sin(6\omega_1 t + \alpha_7), \end{aligned} \quad (33)$$

thus, only the phase of this power affected, but not its frequency.

Let the load generates both the 5th and the 7th order current harmonics, i.e., the load is as shown in Fig. 8.

The oscillating component of the instantaneous power for such a load is

$$\tilde{p} = p_5 + p_7 = 3U_1[I_5\cos(6\omega_1 t + \alpha_5) + I_7\cos(6\omega_1 t + \alpha_7)], \quad (34)$$

while the instantaneous reactive power is

$$\tilde{q} = q_5 + q_7 = 3U_1[-I_5\sin(6\omega_1 t + \alpha_5) + I_7\sin(6\omega_1 t + \alpha_7)]. \quad (35)$$

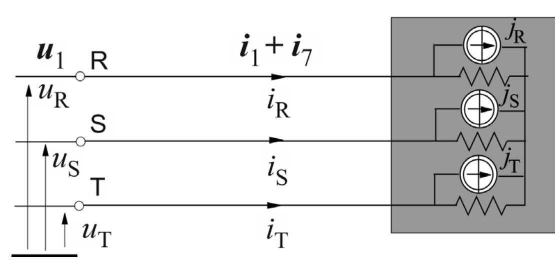

Fig. 7. Resistive balanced HGL generating the 7th order current harmonic

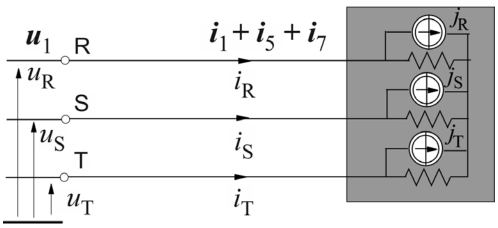

Fig. 8. Resistive balanced HGL generating the 5th and the 7th order current harmonic

Equations (34) and (35) show that oscillating components of instantaneous active and reactive powers are not associated with particular power properties of the load, but only with rms values of harmonics, I_5, I_7, and their phases in α_5 and α_7. In particular, if

$$I_7 = I_5 \quad \text{and} \quad \alpha_5 = \alpha_7, \text{ then} \tag{36}$$

$$\tilde{p} = 2\,p_5 = 6U_1 I_5 \cos(6\omega_1 t + \alpha_5), \tag{37}$$

while if

$$I_7 = I_5 \quad \text{and} \quad \alpha_7 = \alpha_5 + 180^{\text{o}}, \text{ then} \tag{38}$$

$$\tilde{p} \equiv 0, \quad \tilde{q} = 2q_5 = -6U_1 I_5 \sin(6\omega_1 t + \alpha_5) \tag{39}$$

Thus, at such properties, it does not seem that these powers might be associated with any power property of the load.

VI. INSTANTANEOUS POWERS OF RESISTIVE BALANCED LOADS AT NONSINUSOIDAL VOLTAGE

Now, let us supply a resistive balanced load, shown in Fig. 9, with symmetrical voltage distorted by the 5th order harmonic.

Assuming that the voltage at terminal R is

$$u_R = \sqrt{2}\,U_1 \cos\omega_1 t + \sqrt{2}\,U_5 \cos 5\omega_1 t\,, \tag{40}$$

the instantaneous power of the load in such a situation, according to formula (7) is equal to

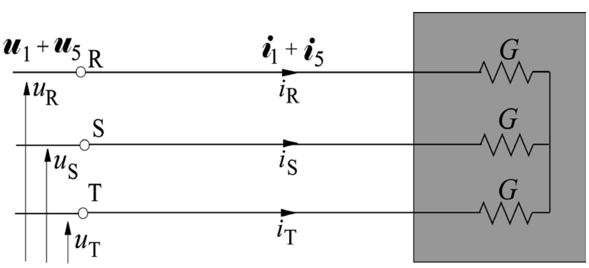

Fig. 9. Resistive balanced load supplied with voltage composed of fundamental and the 5th order harmonic

$$p = \boldsymbol{u}^{\text{T}}\boldsymbol{i} = \boldsymbol{u}^{\text{T}}\, G\, \boldsymbol{u} = G[\boldsymbol{u}_1 + \boldsymbol{u}_5]^{\text{T}}[\boldsymbol{u}_1 + \boldsymbol{u}_5] =$$
$$= G\,\boldsymbol{u}_1^{\text{T}}\boldsymbol{u}_1 + G\,\boldsymbol{u}_5^{\text{T}}\boldsymbol{u}_5 + G(\boldsymbol{u}_1^{\text{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\text{T}}\boldsymbol{u}_1). \tag{41}$$

The first two terms are constant components of the instantaneous power

$$G\,\boldsymbol{u}_1^{\text{T}}\boldsymbol{u}_1 + G\,\boldsymbol{u}_5^{\text{T}}\boldsymbol{u}_5 = G||\boldsymbol{u}_1||^2 + G||\boldsymbol{u}_5||^2 = P. \tag{42}$$

The last term is equal to

$$G(\boldsymbol{u}_1^{\text{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\text{T}}\boldsymbol{u}_1) = G[u_{1\text{R}}\,u_{5\text{R}} + u_{1\text{S}}\,u_{5\text{S}} + u_{1\text{T}}\,u_{5\text{T}}\,] +$$
$$+\ G[u_{5\text{R}}\,u_{1\text{R}} + u_{5\text{S}}\,u_{1\text{S}} + u_{5\text{T}}\,u_{1\text{T}}\,] =$$
$$= 2G[u_{1\text{R}}\,u_{5\text{R}} + u_{1\text{S}}\,u_{5\text{S}} + u_{1\text{T}}\,u_{5\text{T}}\,] =$$
$$= 4GU_1 U_5[\cos\omega_1 t \times \cos 5\omega_1 t +$$
$$+ \cos(\omega_1 t - 120^0) \times \cos(5\omega_1 t + 120^0) +$$
$$+ \cos(\omega_1 t + 120^0) \times \cos(5\omega_1 t - 120^0)] =$$
$$= 6GU_1 U_5 \cos 6\omega_1 t. \tag{43}$$

Consequently, the instantaneous power of the load is

$$p = \bar{p} + \tilde{p} = P + 6GU_1 U_5 \cos 6\omega_1 t. \tag{44}$$

The instantaneous reactive power of such a resistive balanced load is, of course, equal to zero at any instant of time, meaning

$$\tilde{q} \equiv 0 \tag{45}$$

Thus, the load supplied with nonsinusoidal voltage as shown in Fig. 9, does not differ in terms of the IRP p-q Theory from a harmonic generating load as shown in Fig. 8, with sinusoidal supply voltage, assuming that current harmonics satisfy condition (37) and $\alpha_5 = 0$.

After filtering the instantaneous powers *p* and *q* calculated according to IRP p-q Theory, the results for circuits shown in Figures 8 and 9 have the same form, given by formulas (27). Consequently, these two substantially different circuits cannot be distinguished in terms of instantaneous *p* and q powers. This observation was reported in Ref. [16]. This observation is important also for the IRP p-q Theory applications as a control algorithm. Loads in Figures 8 and 9, having similar *p* and *q* powers, are mutually different respective needs and possibilities of their compensation. Similar observation, but with respect to the line currents asymmetry, which could be caused by the load imbalance or the supply voltage asymmetry, was reported in Ref. [17].

VII. CONCLUSIONS

Explanation and description of power properties of electrical loads supplied with a nonsinusoidal voltags and currents is the main objective of the power theory development. It was shown in this paper that a load supplied with a nonsinusoidal voltage may not be distinguished in terms of *p* and *q* powers from a harmonics generating load

(HGL) supplied with a sinusoidal voltage, however. Loads with nonsinusoidal supply voltage can have p and q powers identical to these powers of a substantially differrent HGL at sinusoidal supply voltage. It means that the IRP p-q Theory does not identify power properties of electrical loads supplied with nonsinusoidal voltage. Moreover, there is no difference in some cases between effects of the load originated harmonics on the instantaneous reactive power q and on the oscillating component $\tilde{q}$ of the instantaneous active power.

Also interpretation of the instantaneous reactive power q as a measure of energy rotation around supply lines or exchange between phases, as suggested by authors of the IRP p-q Theory, is not true. Thus, the Instantaneous Reactive Power p-q Theory, widely disseminated in electrical engineering of systems with nonsinusoidal voltages and currents, seems to have very shaky physical fundamentals.

REFERENCES

1. Akagi H., Kanazawa Y., Nabae A.: *Instantaneous reactive power compensator comprising switching devices without energy storage components*. IEEE Trans. IA., IA-20, No. 3, pp. 625–630. 1984.
2. Monteiro L.F., Afonso J.L., Pinto J.G., Watanabe E.M., Aredes M., Akagi H.: *Compensation algorithms based on the p-q and CPC theories for switching compensators in micro-grids*. Proc. of COBEP 09 Power Electronics Conference, Brazilian DOI: 10.1109/ COCEP.2009.5347593, pp. 32–40. 2009.
3. Budeanu C.I.: *Puissances Reactives et Fictives*. Institut Romain de l'Energie, Bucharest. 1927.
4. Fryze S.: *Active, Reactive and Apparent Power in Circuts with Nonsinusoidal Voltages and Currents*. (in Polish) Przegląd Elektrotechniczny, z.7, 193–203, z.8, 225–234, (1932), z.22, 673–676. 1931.
5. Quade W.: *Uber Wechselstrome mit beliebiger Kurvenform in Dreiphasensystemen*. Archiv fur Elektr., 28, 798–809. 1934.
6. Shepherd W., Zakikhani P.: *Suggested Definition of Reactive Power for Nonsinusoidal Systems*. Proc. IEE, 119, No. 9, pp. 1361–1362. 1972.
7. Depenbrock M.: *Wirk- und Blindleistung*. ETG-Fachtagung „Blindleistung," Aachen. 1979.
8. Kusters N.L., Moore W.J.M.: *On the Definition of Reactive Power Under Nonsinusoidal Conditions*. IEEE Trans. Pow. Appl. Syst., PAS-99, No. 3, 1845–185. 1980.
9. Akagi H., A. Nabae: *The p-q theory in three-phase systems under non-sinusoidal conditions*. European Trans. on Electrical Power, ETEP, Vol. 3, No. 1, pp. 27–31. 1993.
10. Willems J.L.: A new interpretation of the Akagi-Nabae power components for nonsinusoidal three-phase situations, IEEE Trans. actions on Instr. and Meas., Vol. 41, pp. 523–527. 1992.
11. Czarnecki L.S.: *Orthogonal decomposition of the current in a three-phase non-linear asymmetrical circuits with nonsinusoidal voltage*. IEEE Trans. Instr. Measur., IM-37, No. 1, pp. 30–34. 1988.
12. Akagi H., Watanabe E. H., Aredes M.: *Instantaneous Power Theory and Applications to Power Conditioning*. John Wiley & Sons. 2007.
13. Czarnecki L.S.: *Could power properties of three-phase systems be described in terms of the Poynting Vector*? IEEE Trans. on Power Delivery, Vol. 21, No. 1, pp. 339–344. 2006.
14. Czarnecki L.S., Ginn H. L.: *The effect of the design method on efficiency of resonant harmonic filters*. IEEE Trans. on Power Delivery, Vol. 20, No. 1, pp. 286–291. 2005.
15. Czarnecki L.S.: *On some misinterpretations of the Instantaneous Reactive Power p-q Theory*. IEEE Trans. on Power Electronics, Vol. 19, No. 3, pp. 828–836. 2004.
16. Czarnecki L.S.: *Effect of supply voltage harmonics on IRP-based switching compensator control*. IEEE Trans. on Power Electronics, Vol. 24, No. 2, pp. 483–488. 2009.
17. Czarnecki L.S.: *Effect of supply voltage asymmetry on IRP p-q - based switching compensator control*, IET Proc. on Power Electronics, Vol. 3, No. 1, pp. 11–17. 2010.
18. Czarnecki L.S.: *Power theories and meta-theory of powers in electrical circuits*. Przegląd Elektrotechniczny (Proc. of Electrical Engineering), R. 87, No. 8/2011, pp. 197–200. 2011.

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