

# Some Idea on Energy Consumption of Non-Linear Consumers

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**Summary:** The determination of energy consumption required by non-linear consumer results in numerous dilemmas. According to some conceptions the energy consumed by non-linear consumers ought to be calculated or measured by taking the favoured harmonic components only into considerations instead of the all harmonic content of voltage and current supplying the non-linear consumer. The paper shows how to change the value of energy calculated by the full harmonic content of the measured voltage and current, respectively, e.g. by the fundamental harmonic of voltage in the case of the most typical consumer, the two-wave connection rectifier with buffer condenser as non-linear load. It is especially interesting for practical arrangements where non-linear consumers resulting in similar network pollution and rectifier loads are connected parallel to the same line.

**Key words:** non-linear loads, power quality, power measuring problems

## 1. INTRODUCTION

In our days the decisive part of electric energy is used up by apparatuses polluting the network significantly due to their non-linear features. Especially, the number of the electronic equipment connecting to the public utility line increases. They are as follows: first generation entertainment apparatuses, computer means, notebook and mobile phone accumulator chargers, compact fluorescent lamps, controllable household appliances, welding apparatus, etc. A similar problem arises in the field of industrial consumers, e.g. controlled electric drives. These equipment, generally, require intermittent and filtered DC voltage produced from the AC line by discontinuous current conduction mode, their input circuits are one- or three-phase diode rectifiers and buffer condensers. This means that the rectifiers operate as “peak-rectifiers”. The reason of non-linear quality is that energy feeding to buffer condenser occurs in short pulses around the peak value of line voltage (Fig. 1.).

Nowadays, the wave-form distortion of line voltage surpasses the allowable degree because of the non-linear quality of consumers. The network pollution is not only the reason of serious problems increasing the line and consumer losses but creates the source of breakdowns and line drop-out resulting in over-voltage at switching, over-heating and electromagnetic noises, as well. The network pollution significantly decreases the reliability of line, furthermore, it causes numerous measuring problems of electric quantities. The problem sphere referring to the mitigation and improvement of network pollution has been discussed at many conferences and periodicals

during the past years. The authors’ proposals are published in [2, 3, and 4]. Our proposal was to apply separate low voltage DC distribution at consumers’ where the number of electronic equipment polluting the line is high. The separate DC line is connected to the public utility line through a network-friendly converter. In the paper the emphasis is on measuring technique problems provided that the present situation does not change. According to some conceptions the energy consumed by non-linear consumers ought to be calculated or measured by taking only the favoured harmonic components into considerations instead of the all harmonic content of voltage and current supplying the non-linear consumer.

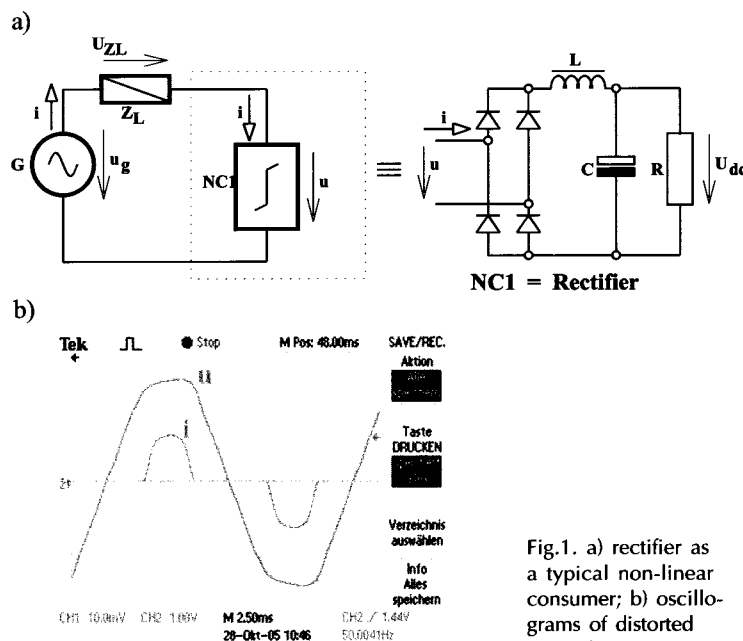


Fig.1. a) rectifier as a typical non-linear consumer; b) oscillograms of distorted wave-forms

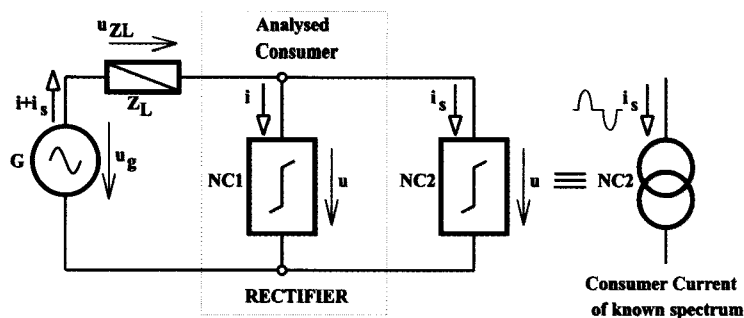


Fig. 2. Arrangement for analysing the mutual effects of non-linear consumers

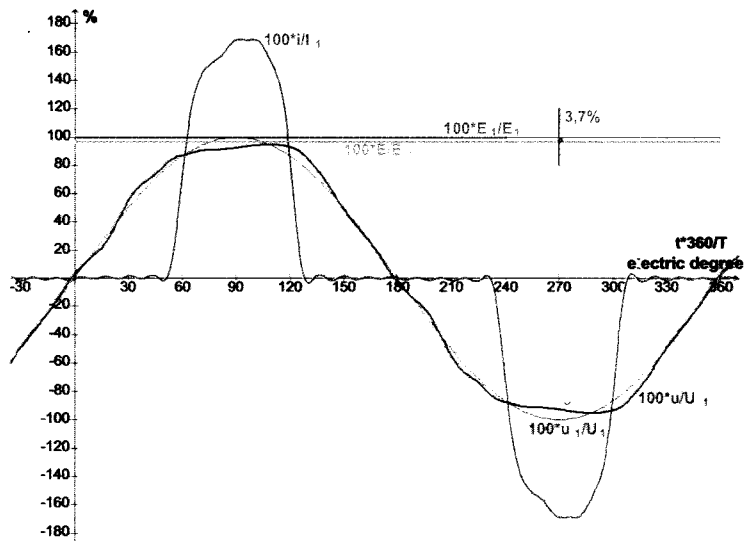


Fig. 3. Network distortion of welding equipment

## 2. PROBLEM DESCRIPTION

The question is how to form the value of energy calculated by the full harmonic content of the measured voltage and current, respectively, e.g. the fundamental harmonic of voltage in the case of non-linear consumer NC1 (two-wave rectifier with RC load) shown in Figure 1a if the wave-forms on the terminals are similar to those in Figure 1b.

The case in Figure 2, where another, non-linear consumer NC2 resulting similar network pollution is connected parallel to the analysed non-linear consumer NC1 with the impedance  $Z_L$  of line, is more interesting. The consumers are supplied by generator G of pure sinusoidal voltage or voltage of full harmonics  $u_g$ . The most typical non-linear consumer used in large number is the two-wave connection rectifier with buffer condenser. This is the cause why we analyse this kind of load in details.

## 3. ENERGY ON CONSUMER TERMINALS

Figure 3 shows the wave-form of the current and supply voltage of the welding equipment consisting of a bridge connection rectifier and buffer condenser, taking the 1–25 Fourier odds

harmonics into consideration. The spectrum was recorded by an oscilloscope of power analyzer from the evaluation of oscillogram in Figure 1b. Energy  $E$  may be calculated through the analytical method from the product of harmonic components of the terminal voltage  $u(t)$  and line current  $i(t)$ .

$$E = \int_0^T [u_1(t) + u_3(t) + u_5(t) + \dots] \cdot [i_1(t) + i_3(t) + i_5(t) + \dots] dt \quad (1)$$

where:

$u(t) = [u_1(t) + u_3(t) + u_5(t) + \dots]$  — is the terminal voltage;  
 $i(t) = [i_1(t) + i_3(t) + i_5(t) + \dots]$  — is the line current.

Furthermore, energy  $E_1$  may also be determined from the product of harmonic components of the fundamental harmonic voltage  $u_1(t)$  and current  $i(t)$ .

$$E_1 = \int_0^T u_1(t) \cdot [i_1(t) + i_3(t) + i_5(t) + \dots] dt = \int_0^T u_1(t) \cdot i_1(t) dt \quad (2)$$

where:

$u_1(t)$  — is the fundamental harmonic voltage;  
 $i_1(t)$  — is the fundamental harmonic of line current.

Based on the equations, the ratio of  $E/E_1$  and the periodical time  $T$  of fundamental harmonic can be calculated. According to the measured oscillogram,  $E_1$  is larger than  $E$  by 3.7%. Regarding the above stating, the real value of energy consumption is problematic on the output of rectifier. It is trivial but we have to state whether the energy  $E$  or  $E_1$  is the valid.

## 4. ENERGY USED BY RECTIFIERS AT SINUSOIDAL AND DISTORTED LINE VOLTAGE

It is not difficult to calculate the energy used on the terminals of a rectifier, assuming that the rectifier is loss-free. Energy  $E_e$  used by the loading resistor has been investigated by modelling the arrangement in Figure 1a:

$$E_e = \int_0^T \frac{U_e^2(t)}{R} dt \quad (3)$$

where:

- $U_e(t)$  — is the dc voltage;
- $R$  — is the loading resistor.

Furthermore, we have calculated the values of energy  $E$  and  $E_1$ , besides, their ratio in sinusoidal and distorted voltage supplies, as well. The line impedance was assumed to be of zero value. In sinusoidal supply the voltage of generator  $G u_g$  was equivalent to the fundamental component of voltage  $u_1$  ( $u_g = u_1$ ), see in Figure 3.

In distorted supply the voltage  $u$  of generator  $G u_g$  contained all the harmonics in addition to the fundamental harmonic  $u_1$  ( $u_g = u$ ). The harmonic components are the consequence of the distorting effect of the wave-form produced by adjoining consumers on the line impedance. Consequently, both analysed cases are based on measuring results. The energy on dc side ( $138.7\text{Ws} \div 6935\text{W}$ ) was held constant by choosing the appropriate value of resistor  $R$  while the value of buffer condenser influencing the ripple of dc voltage was changed.

Figure 4 refers to sinusoidal voltage supply, while Figure 5 is valid for distorted voltage supply. The diagrams show energy  $E$ ,  $E_1$ ,  $E_e$ , furthermore, the dc voltage  $U_E$  and its ripple content  $\Delta U_E$ , the value of load resistor  $R$ , the peak  $i_{peak}$  and effective  $I_{eff}$  value of current  $i$ , the effective value of fundamental current  $I_1$  of current  $i$ , the quotient  $I_{eff}/I_1$  in steady state condition referring one-one period, which is the reciprocal value of the distortion factor. The value of buffer condenser influences the waveform of current  $i$ . This is why the dim diagrams in different range are drawn in Figures 4 and 5. They represent the different, characteristic current waveforms varying in the function of condenser value.

In sinusoidal supply the identity  $E = E_1 = E_e$  is trivial because the fundamental harmonic of supply voltage  $u_1(t)$  can create power with the fundamental harmonic of current  $i_1(t)$  only, see Equation 2. The rectifier is assumed to be loss-free, consequently, energy  $E_e$  is the same as the others (see Figure 4, where the deviation 0.57% can be explained by neglecting the harmonic components of higher than 25). In distorted voltage line supply the situation is fundamentally different, as the relation  $E = E_e \ll E_1$  will be true, depending on the value of buffer condenser, see in Figure 5. As a matter of fact, voltage and current harmonics with the same number of order can create effective power but the sum of power of upper harmonics is summarised with the power of fundamental harmonic in one range ( $0.23\text{ mF} < C < 1\text{ mF}$ ), while it is subtracted from the power of

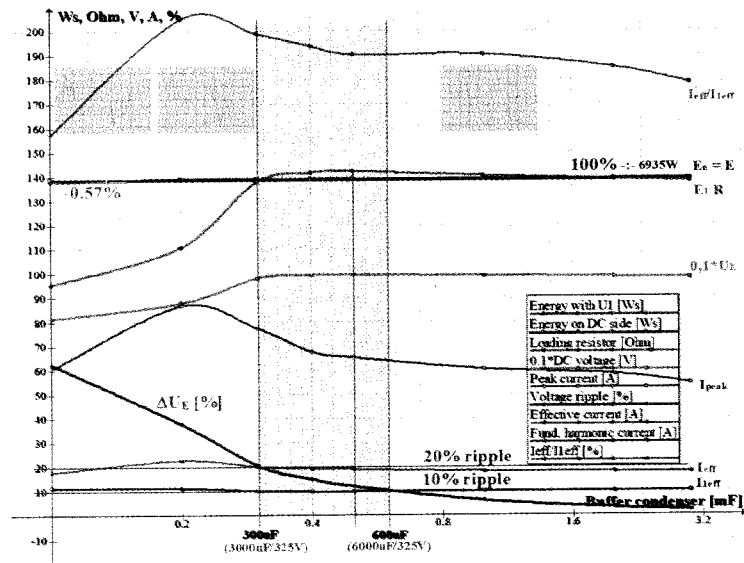


Fig. 4. Rectifier supplied by sinusoidal voltage

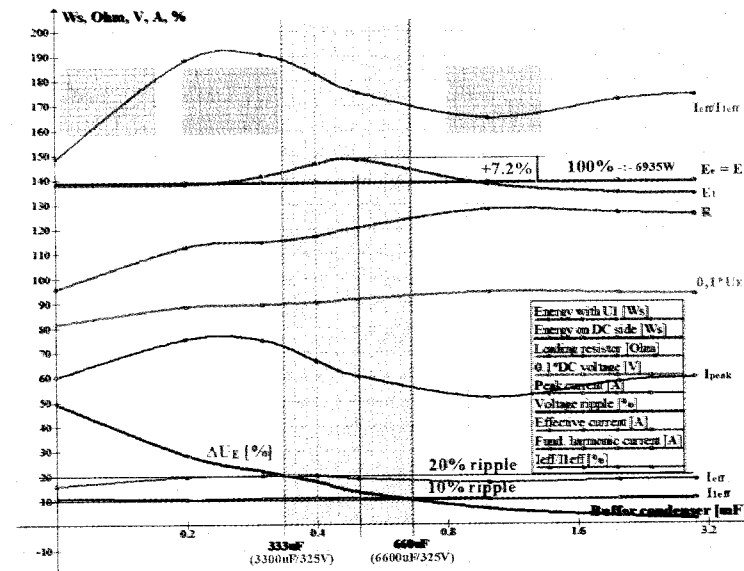


Fig. 5. Rectifier supplied by distorted voltage

fundamental harmonic in other range ( $C < 0.23\text{ mF}$ ,  $C > 1\text{ mF}$ ). The area marked with slash lines shows the allowable ripple range (10–20%) of dc voltage  $\Delta U_E$  in the practice. It is surprising that energy  $E_1$  may be larger by 7.2% than the used energy  $E_e$  in the range of interest.

## 5. THE EFFECT OF ADJOINING CONSUMER TO THE ENERGY INPUT OF RECTIFIER

The mutual effect of consumers due to the line impedance  $Z_M = R_M$  has been analysed by modelling the arrangement in Figure 2. The presence of adjoining non-linear consumer NF2 has been modelled by current  $i_s$  of known spectrum from measurement. The value of buffer condenser of the rectifier was assumed 0,5mF.

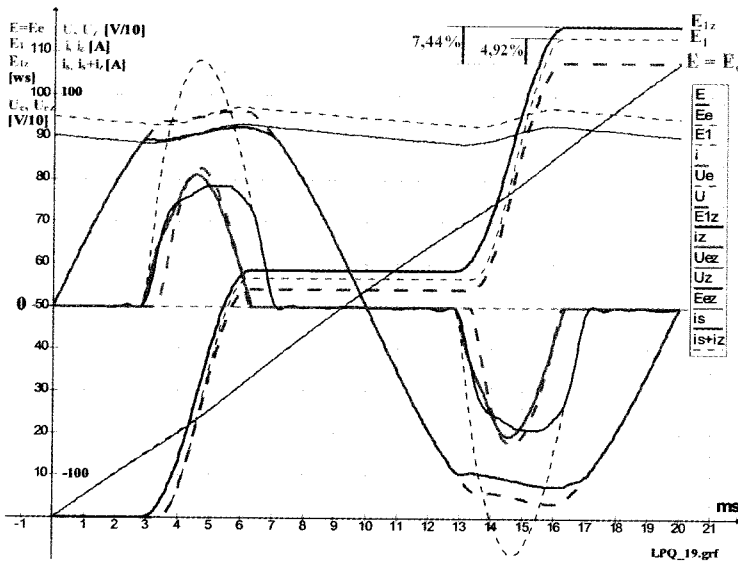
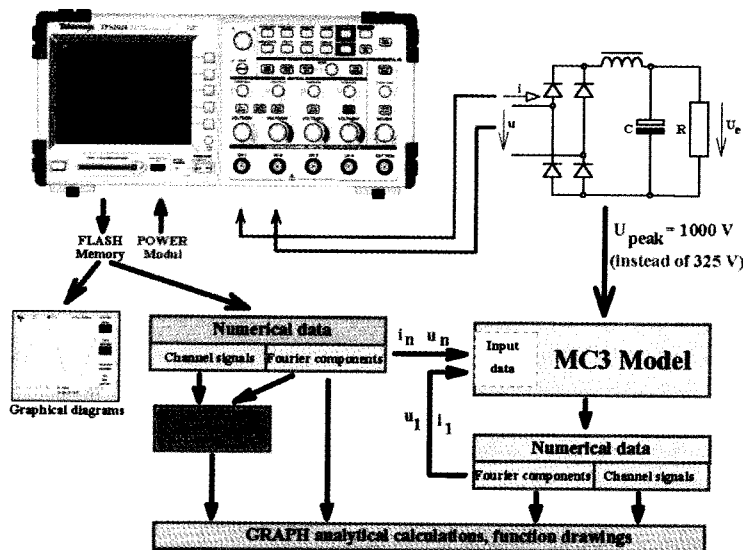


Fig. 6. Wave-form distortions and energy relations without and with adjoining consumer

This value is shown with a vertical line in Figures 4 and 5. The modelling has been done by currents  $i_s = 0$  and  $i_s > 0$ , see the curves with discontinuous, respectively, continuous ( $z$  index) lines in Figure 6. The dc energy  $E_e$  was constant in course of modelling.

The modelling results in Figure 6 show the following quantities in the steady state condition and function of time: terminal voltage  $U$  and  $U_z$  of consumers, current input  $i$  and  $i_z$  of analysed consumer NF1, current  $i_s$  of adjoining NF2, the common current  $i_s + i_z$  in the line impedance, and the energies calculated by equations (1)–(3). It can be seen that energy  $E$  calculated by Equation 1, respectively,  $E_z$  in presence of the adjoining non-linear consumer is equivalent to  $E_e$  even if the stroke of curves is different. It is natural that the energy drawn up from the line on the ac input side increases

Fig.7. Method of calculating and modelling procedure



in every half-period by surge quantum when current is not zero. The current of loading resistor on the output dc side flows continuously; therefore, the energy consumption increases near linearly on the output. At the end of one-period energy  $E$  (in the case of adjoining non-linear consumer  $E_z$ ) is always equivalent to  $E_e$ . We repeatedly emphasize that energy  $E_e$  has been held constant. The above assumption corresponds to the state when the rectifier is a part of the controlled supply unit providing constant power.

## 6. METHOD OF CALCULATING AND MODELLING PROCEDURE

In the following space the process of calculating and modelling procedures will be discussed briefly. The results shown earlier are based on this procedure.

The calculation results in Figure 3 come from the measurements of electronic welding equipment containing rectifier according to Figure 1a. The wave-forms of line voltage  $u$  and input current  $i$  from the line are taken up by an oscilloscope with power module in the arrangement in Figure 7. The picture format (TIF) and sampled numerical data (channel signals 2500 sample/period in CSF format) of quantities were taken up by the oscilloscope and stored in its FLASH memory. The memory stores also the data of the Fourier components of measured wave-forms calculated by the power module (amplitude, phase angle, TRMS values, etc. of 1–50 harmonic components in CSV format, at the calculations and modelling the harmonic components of ordinal number 1–25 were used). For processing the channel signals and Fourier components, the EXCELL calculation tables provided useful means by producing the instruction lines of the represented functions for drawing the wave-forms, respectively, analytical calculations of the program GRAPH. Based on the above, the integral calculations necessary for stating the energy ( $E_1, E_{1n}, E_{nn}$  in Figure 3) are done by the program GRAPH in an analytical way.

The calculation of energy on the ac input side and dc output side of the rectifier supplied by sinusoidal, respectively, distorted voltage is done by modelling. The model constructed by the program MC3 corresponds to the circuit diagram in Figures 1a and 7. The model was assumed to contain ideal diodes (minimal conduction voltage drop, zero value of conduction resistance, furthermore, leakage current and charge storage). In the interest of neglecting some tenths voltage drop, the rectifier model was supplied with voltage of

peak value 1000V higher than that of the public utility line. By this the voltage drop in conduction mode may be neglected, in fact. In the calculations regarding Figure 4, the input voltage of the rectifier model was ideal sinusoidal ( $u_g = u_1$ ) corresponding to the fundamental harmonic voltage. In the calculations regarding Figure 5, the input voltage of the rectifier model was the distorted voltage ( $u_g = u$ ) of welding equipment taken up by oscilloscope measurement. The modelling program MC3 can also store the wave-forms and results from the Fourier analysing routine in file format CSV, in addition to the usual monitor pictures. The quantities from the Fourier analysis, e.g. the effective values of  $u$  and  $i$ , will be repeatedly processed at every program running. The function drawing program GRAPH was used for the illustration of modelling results with different parameters.

The effect of adjoining non-linear load to the power consumption has been analysed by the modified rectifier model shown in Figure 2. A given proportion of line current  $i_S$  measured by oscilloscope indicating the adjoining disturbances. The Fourier components of current  $i_S$  have been determined earlier. Regarding the results in Figure 6, the model has been supplied by sinusoidal fundamental harmonic voltage with line impedance  $Z_M = R_M$ . In course of the modelling procedures  $i_S = 0$  relation is valid in independent operation mode but in the case of adjoining disturbances,  $i_S$  is higher than zero. For the elaboration and graphical representation of measuring results, the function drawing program GRAPH was used in the described way, see Figure 7.

It is worth pointing out that the investigations employing modelling procedure are based on measurements done by only one up-to-date appliance in all the three cases. In the first case (Fig. 3.) the results of energy calculations completed by analytical method are based on measurements only. In the second and third cases the model supplied by measuring data provides results based on the measured voltage values in Figures 4 and 5, respectively, current values in Figure 6. Consequently, the model can be regarded a special calculation routine.

## 7. CONCLUSIONS

The completed analyses verify unambiguously that energy  $E$  drawn up by the non-linear quality ideal rectifier load from the line calculated by Equation 1 is equal with the used up energy  $E_e$  calculated by Equation 3 in all cases. But energy  $E_1$  calculated by Equation 2 is

larger than energy  $E$  and  $E_e$  in practical cases. It is noteworthy that energy  $E_1$  calculated by the fundamental harmonic voltage  $u_1$  is larger by 4.92% than the used up energy  $E_e$  in the examined independent working condition but the deviation will be 7.44% if there is an adjoining non-linear consumer on the same line, while the consumption of the analysed consumers is the same in both cases. The shown modelling procedure based on measuring results may reflect some problems concerning the true and real measurement of energy consumption. This problem sphere is discussed in details in [5].

Our earlier publications [2, 3, 4] demonstrated in many cases that the effective value  $I_{eff}$  of loading current  $i$  of high harmonic content created by rectifiers significantly surpasses the effective value  $I_{1eff}$  of fundamental harmonic current  $i$ . Based on the diagrams in Figures 4 and 5 the proportion of  $I_{eff}/I_{1eff}$  is equivalent to 1.7–1.95 in the range of interest. This fact results in over-dimensioning the cross-section of wires of public utility line for ensuring the reliability of energy supply at significant rectifier loading. For avoiding the above detriment, the application of network-friendly rectifiers or pre-converters is preferred in electronic consumers. Our earlier suggestion [2, 3, 4] regarding the cost reduction and other important viewpoints preferred the application of common network-friendly rectifier for the consumer groups chosen suitably.

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