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A METHOD FOR DETERMINING ELECTRIC QUANTITIES IN A WORKPIECE HEATED BY INDUCTION

Metoda wyznaczania wielkości elektrycznych we wsadzie nagrzewanym indukcyjnie

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Summary: Assuming constancy of magnetic flux induced in a long coil under both load and no-load condition makes it easier to calculate magnetic field intensity H_{mo} at the surface of a workpiece heated by induction. A knowledge of H_{mo} makes it possible to analyse various electric quantities in the workpiece. As an example, the real and reactive components of power in the workpiece have been analysed. Conditions that must be met to allow replacing the cylindrical electromagnetic wave with a plane wave have also been given.

Streszczenie: Przyjęcie stałości strumienia magnetycznego wytworzonego w długim wzbudniku zarówno pustym, jak i ze wsadem, ułatwia obliczenie pola magnetycznego H_{mo} na powierzchni nagrzewanego indukcyjnie wsadu. Znajomość H_{mo} umożliwia zanalizowanie różnych wielkości elektrycznych występujących we wsadzie. Przykładowo zanalizowano moce czynną i bierną wsadu. Podano też warunki umożliwiające zastępowanie fali elektromagnetycznej walcowej przez falę płaską.

Keywords: electroheat, induction heating, electromagnetic field

Słowa kluczowe: elektrotermia, nagrzewanie indukcyjne, pole elektromagnetyczne

1. INTRODUCTION

The presented method for determining electric quantities in a workpiece heated by induction is based on the approximate equality of magnetic flux magnitudes in the coil under both load and no-load condition. The method is valid for a long cylindrical coil supplied with a sinusoidally varying current, inducing a cylindrical electromagnetic wave in a heated workpiece of circular cross-section. An induction heater with such a coil has a low power factor of the order 0.1. Therefore, the EMF induced in a w-turn exciting coil, $E = 4.44\ \Phi_m f w$, practically equals the voltage U supplying the coil with load. If, e.g., the reactance of the heater is 10 times the resistance, then $E = 0.995U$.

The magnetic flux amplitudes, Φ_m, in an empty coil and in the coil with load are therefore practically equal. The amplitude of magnetic field intensity, H_{mo}, at the surface of the workpiece can be computed in the way described in [2], basing on equality of the above flux magnitudes. A knowledge of H_{mo} makes it possible to analyse various electric quantities in the heated workpiece.

The following considerations are limited to determining the real and reactive components of power in the workpiece. In addition, the dependence of the above mentioned power components on the coil current frequency and on the width of the air gap between the coil and the workpiece have been discussed. The power dissipated in the workpiece, computed on the assumption that a cylindrical electromagnetic wave spreads in the workpiece has also been compared with the power obtained by replacing the cylindrical wave with a plane one.

2. DETERMINATION OF MAGNETIC FIELD INTENSITY AT THE SURFACE OF A WORKPIECE

In an empty coil of internal radius r_1, length $h \rangle\rangle 2r_1$ and inductance:

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$$L = \frac{\pi r_1^2 \mu_0 w^2}{h} \qquad (1)$$

the current, due to the voltage U, of amplitude

$$I_m = \frac{1}{\sqrt{2}\pi^2 \mu_0} \frac{hU}{f r_1^2 w^2} \qquad (2)$$

alternating at an angular frequency $\omega = 2\pi f$ induces magnetic flux of peak value

$$\Phi_m = \frac{1}{\sqrt{2}\pi} \frac{U}{fw} \qquad (3)$$

After placing a workpiece of radius r_2, length $h \rangle\rangle r_2$, elevtric conductivity γ and magnetic permeability $\mu = \mu_0 \mu_r$ into the coil, the magnetic flux will be a sum of the flux in the air-gap

$$\Phi_{mp} = \pi \mu_0 (r_1^2 - r_2^2) H_{mo} \qquad (4)$$

and the flux $\underline{\Phi}_{mw}$ penetrating the workpiece. A cylindrical electromagnetic wave is spreading in the workpiece and the distributions of magnetic field, $\underline{H}_m$, and electric field, $\underline{E}_m$, along radius r of the workpiece are given by [1, 2, 3]:

$$\underline{H}_m = H_{mo} \frac{J_0\left(\sqrt{2}\frac{r}{\delta} j^{\frac{3}{2}}\right)}{J_0\left(\sqrt{2}\frac{r_2}{\delta} j^{\frac{3}{2}}\right)} = H_{mo} \frac{ber\left(\sqrt{2}\frac{r}{\delta}\right) + jbei\left(\sqrt{2}\frac{r}{\delta}\right)}{ber\left(\sqrt{2}\frac{r_2}{\delta}\right) + jbei\left(\sqrt{2}\frac{r_2}{\delta}\right)} \qquad (5)$$

$$\underline{E}_m = j^{\frac{3}{2}} \frac{\sqrt{2}}{\gamma\delta} H_{mo} \frac{J_1\left(\sqrt{2}\frac{r}{\delta} j^{\frac{3}{2}}\right)}{J_0\left(\sqrt{2}\frac{r_2}{\delta} j^{\frac{3}{2}}\right)} = -\frac{\sqrt{2}}{\gamma\delta} H_{mo} \frac{ber'\left(\sqrt{2}\frac{r}{\delta}\right) + jbei'\left(\sqrt{2}\frac{r}{\delta}\right)}{ber\left(\sqrt{2}\frac{r_2}{\delta}\right) + jbei\left(\sqrt{2}\frac{r_2}{\delta}\right)} \qquad (6)$$

where:

$J_0\left(\sqrt{2}\frac{r}{\delta} j^{\frac{3}{2}}\right)$ and $J_1\left(\sqrt{2}\frac{r}{\delta} j^{\frac{3}{2}}\right)$ — Bessel functions of the first kind order zero and one with complex argument $\sqrt{2}\frac{r}{\delta} j^{\frac{3}{2}}$, [4]

$\delta = \sqrt{\frac{1}{\pi f \gamma \mu}}$ — depth of penetration of electromagnetic wave, $j^{\frac{3}{2}} = \sqrt{-j} = \pm\frac{1-j}{\sqrt{2}}$.

The magnetic flux $\underline{\Phi}_{mw}$ determined from

$$\underline{\Phi}_{mw} = \pi\mu\delta^2 \int_0^{x_2} x \underline{H}_m dx \qquad \left(x = \sqrt{2}\frac{r}{\delta} \text{ and } x_2 = \sqrt{2}\frac{r_2}{\delta}\right)$$

is, after substituting eq. (5) and making appropriate calculations and rearrangements, given by

$$\underline{\Phi}_{mw} = \sqrt{2}\pi\mu r_2 \delta H_{mo} (q_1 - jp_1) \qquad (7)$$

where:

$$p_1 = \frac{ber\left(\sqrt{2}\frac{r_2}{\delta}\right) ber'\left(\sqrt{2}\frac{r_2}{\delta}\right) + bei\left(\sqrt{2}\frac{r_2}{\delta}\right) bei'\left(\sqrt{2}\frac{r_2}{\delta}\right)}{ber^2\left(\sqrt{2}\frac{r_2}{\delta}\right) + bei^2\left(\sqrt{2}\frac{r_2}{\delta}\right)} \qquad (8)$$

$$q_1 = \frac{ber\left(\sqrt{2}\frac{r_2}{\delta}\right) bei'\left(\sqrt{2}\frac{r_2}{\delta}\right) - ber'\left(\sqrt{2}\frac{r_2}{\delta}\right) bei\left(\sqrt{2}\frac{r_2}{\delta}\right)}{ber^2\left(\sqrt{2}\frac{r_2}{\delta}\right) + bei^2\left(\sqrt{2}\frac{r_2}{\delta}\right)}$$

The plots of p_1 and q_1 as functions of $\frac{r_2}{\delta}$ are shown in Figure 1.

The amplitude of the total flux Φ_m penetrating the coil with the workpiece equals

$$\Phi_m = \left|\Phi_{mp} + \underline{\Phi}_{mw}\right| = \pi\, r_2^2 \mu_o H_{mo} K \qquad (9)$$

where

$$K = \sqrt{\left[\left(\frac{r_1}{r_2}\right)^2 - 1 + \sqrt{2}\mu_r \frac{\delta}{r_2} q_1\right]^2 + 2\left(\mu_r \frac{\delta}{r_2} p_1\right)^2} \qquad (10)$$

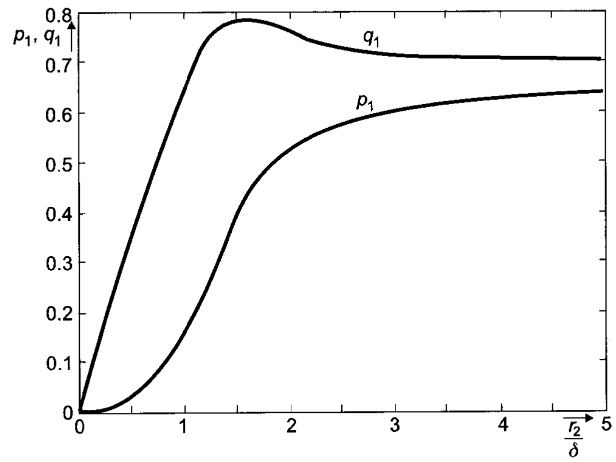

Fig. 1. Plots of $p_1 = f\left(\frac{r_2}{\delta}\right)$ and $q_1 = f\left(\frac{r_2}{\delta}\right)$.

The magnetic field intensity H_{mo} at the workpiece surface can be derived from the assumed equality of magnetic flux magnitudes Φ_m (eqs. (3) and (9))

$$H_{mo} = \frac{1}{\sqrt{2}\pi} U \frac{\gamma\mu_r}{w} \left(\frac{\delta}{r_2}\right)^2 \frac{1}{K} \tag{11}$$

The value of H_{mo} depends also on the ratio $\frac{r_2}{\delta}$.

When the workpiece radius r_2 is very much larger than the depth of penetration δ, the cylindrical wave can be replaced with a plane one in technical calculations. The magnetic and electric field intensities of a plane wave vary against distance as follows [2, 3, 5]

$$\begin{gathered} \underline{H}'_m = H'_{mo}\, e^{-kx}, \quad k = \frac{1+j}{\delta} \\ \underline{E}'_m = \frac{k}{\gamma} H'_{mo}\, e^{-kx} \end{gathered} \tag{12}$$

where distance x is measured from the workpiece surface along its radius. The amplitude of the magnetic flux penetrating the workpiece

$$\underline{\Phi}'_{mw} = \int_o^{r_2} 2\pi r \mu \underline{H}'_m \, dr$$

calculated using eq. (12), identities $r = r_2 - x$, $dr = -dx$ and an approximate equality $1 - e^{-\frac{r_2}{\delta}(1+j)} \approx 1$ is given by

$$\underline{\Phi}'_{mw} = \pi\, r_2 \delta \mu H'_{mo} \left[1 - j\left(1 - \frac{\delta}{r_2}\right)\right] \tag{13}$$

The modulus of the amplitude of the total magnetic flux penetrating the coil with the workpiece has the value

$$\Phi'_m = \left|\Phi_{mp} + \underline{\Phi}'_{mw}\right| = \pi\, r_2^2 \mu_o H'_{mo} K' \tag{14}$$

where

$$K' = \sqrt{\left[\left(\frac{r_1}{r_2}\right)^2 - 1 + \mu_r \frac{\delta}{r_2}\right]^2 + \mu_r^2 \left(\frac{\delta}{r_2} - \frac{\delta^2}{r_2^2}\right)^2} \tag{15}$$

Equating the magnetic flux magnitudes Φ_m (3) and Φ'_m (14) gives the following value of magnetic field intensity H'_{mo} at the surface of a workpiece penetrated by an approximately plane electromagnetic wave

$$H'_{mo} = \frac{1}{\sqrt{2}\pi} U \frac{\gamma\mu_r}{w} \left(\frac{\delta}{r_2}\right)^2 \frac{1}{K'} \tag{16}$$

The expressions for H_{mo} (11) and H'_{mo} (16) differ only with the values of K (10) and K' (15), depending on ratios $\frac{r_1}{r_2}$, $\frac{r_2}{\delta}$ and on μ_r.

A knowledge of H_{mo} or H'_{mo} makes it possible to analyse various electric quantities in the workpiece heated by induction, such as distributions of magnetic field intensity H_m (5), H'_m (12), electric field intensity E_m (6), E'_m (12) as well as those of current and power density. As an example, the workpiece real power, P, and reactive power, Q, will be analysed.

3. REAL AND REACTIVE POWER IN THE WORKPIECE

Surface densities of real power p_{so} and reactive power q_{so} penetrating the workpiece computed on the assumption that a cylindrical electromagnetic wave is spreading in the workpiece (eqs. (5) and (6)) equal

$$\begin{gathered} p_{so} + jq_{so} = -\frac{1}{2} \underline{E}_{mo} H_{mo} = \\ = \frac{H_{mo}^2}{\sqrt{2}\gamma\delta} \frac{ber'\left(\sqrt{2}\frac{r_2}{\delta}\right) + jbei'\left(\sqrt{2}\frac{r_2}{\delta}\right)}{ber\left(\sqrt{2}\frac{r_2}{\delta}\right) + jbei\left(\sqrt{2}\frac{r_2}{\delta}\right)} \end{gathered} \tag{17}$$

Multiplying this identity by the side surface area of the workpiece and substituting eqs. (8) and (11), we obtain an equation for workpiece real power P and reactive power Q

$$P + jQ = \frac{1}{\sqrt{2}\pi} U^2 \frac{h}{w^2} \gamma\mu_r^2 \left(\frac{\delta}{r_2}\right)^3 \frac{1}{K^2} (p_1 + jq_1) \tag{18}$$

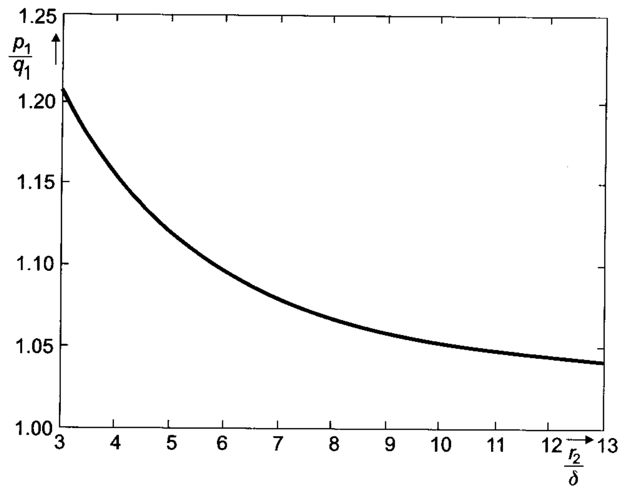

Fig. 2. Curve of $\frac{Q}{P}=\frac{q_1}{p_1}=f\left(\frac{r_2}{\delta}\right)$ for μ_r = 1.

It results from eqs. (18) and (8) that the ratio of reactive power Q to real power P depends only on the ratio $\frac{r_2}{\delta}$

$$\frac{Q}{P}=\frac{q_1}{p_1}=f\left(\frac{r_2}{\delta}\right)$$

This dependence is shown graphically in Figure 2. It can be seen from it that the reactive power is always larger than the real power, and when changing $\frac{r_2}{\delta}$ from 3 to 10, an approximate equality $Q \approx (1.21 \div 1.05)\,P$ occurs.

Let

$$A=\frac{\sqrt{2}\pi\left(\frac{r_2}{\delta}\right)^3 K^2}{p_1}, \qquad B=\frac{\sqrt{2}\pi\left(\frac{r_2}{\delta}\right)^3 K^2}{q_1} \tag{19}$$

We can rewrite eq. (18) as

$$P=\frac{h}{w^2}\gamma\mu_r^2\frac{U^2}{A}, \qquad Q=\frac{h}{w^2}\gamma\mu_r^2\frac{U^2}{B} \tag{20}$$

For loads with μ_r=1 the values of A and B depend only on the ratios $\frac{r_2}{\delta}$ and $\frac{r_1}{r_2}$, and at a fixed workpiece radius r_2, the coil radius r_1 and skin depth δ, depending on the coil current frequency, can be chosen as variables. The values of A and B have been shown graphically in Figures 3 and 4 as a function of $\frac{r_2}{\delta}$ for various ratios $\frac{r_1}{r_2}$. The curves show that at $\frac{r_2}{\delta}$ = const the powers P and Q decrease very rapidly with

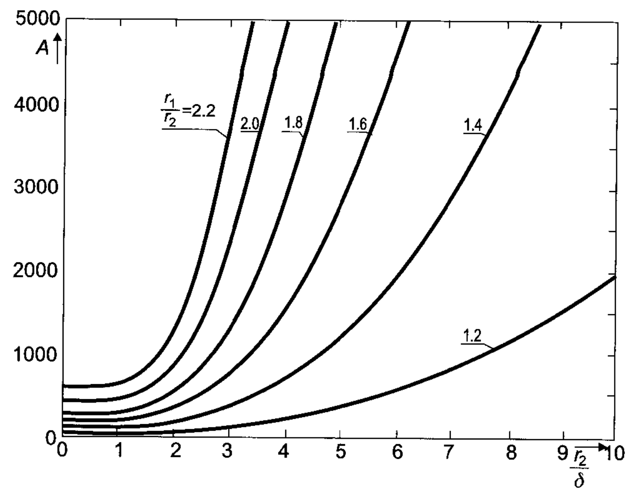

Fig. 3. Plots of $A=f\left(\frac{r_2}{\delta},\frac{r_1}{r_2},\mu_r=1\right)$.

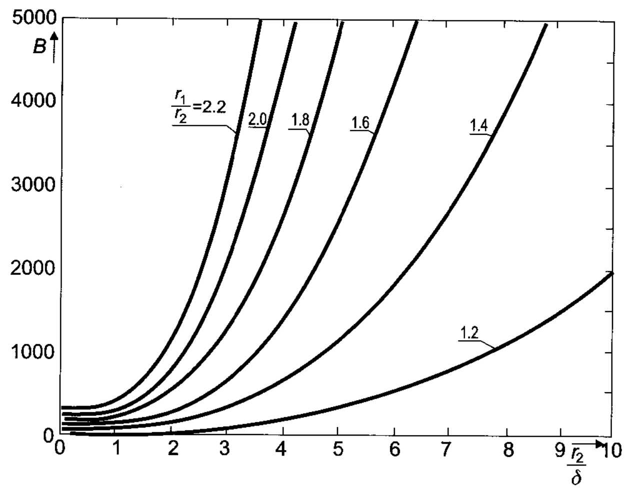

Fig. 4. Plots of $B=f\left(\frac{r_2}{\delta},\frac{r_1}{r_2},\mu_r=1\right)$.

increase of $\frac{r_1}{r_2}$. Also, at $\frac{r_1}{r_2}$ = const and increasing $\frac{r_2}{\delta}$ the powers P and Q decrease, and the more rapidly, the larger the value of $\frac{r_1}{r_2}$. The above dependence results from the fact that at a fixed voltage U supplying the coil, a rise of the coil current frequency f, causing an increase of $\frac{r_2}{\delta}$, decreases the coil current magnitude.

While maintaining a fixed voltage U the powers P and Q depend strongly on the width of the air-gap between the coil and the workpiece.

Assuming a simplification that the load is penetrated by a plane electromagnetic wave (eqs. 12), the surface real power density p'_{so} and reactive power density q'_{so} are given by

$$p'_{so}+jq'_{so} = \frac{1+j}{2\gamma\delta}\left(H'_{mo}\right)^2 \tag{21}$$

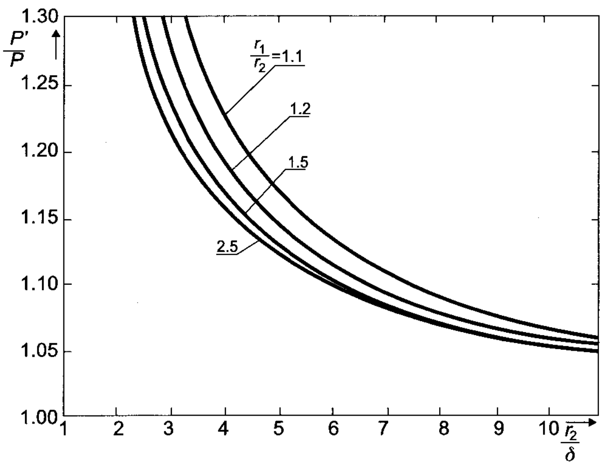

Fig. 5. Plots of $\frac{P'}{P}=f\left(\frac{r_2}{\delta},\frac{r_1}{r_2},\mu_r=1\right)$.

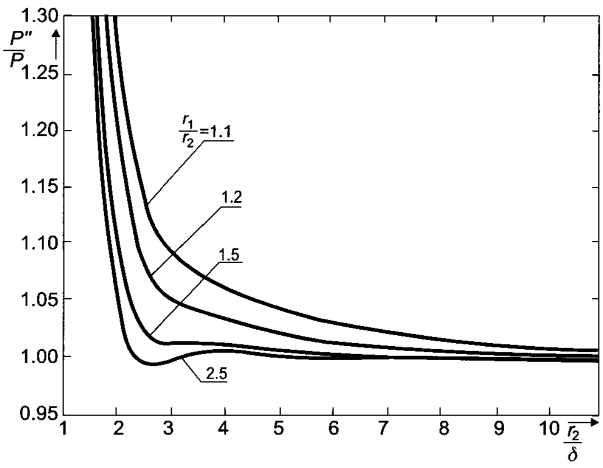

Fig. 6. Plots of $\frac{P''}{P}=f\left(\frac{r_2}{\delta},\frac{r_1}{r_2},\mu_r=1\right)$.

Multiplying this identity by the workpiece side surface area and substituting eq. (16) we obtain an equation for the workpiece real power P' and reactive power Q'

$$P'=Q'=\frac{1}{2\pi}U^2\frac{h}{w^2}\gamma\mu_r^2\left(\frac{\delta}{r_2}\right)^3\frac{1}{(K')^2} \qquad (22)$$

When calculating the workpiece power it is sometimes assumed that its side surface area $S''=2\pi\left(r_2-\frac{\delta}{2}\right)h$ is smaller than the actual one. Hence, we obtain

$$P''=Q''=\frac{1}{2\pi}U^2\frac{h}{w^2}\gamma\mu_r^2\left(1-\frac{\delta}{2r_2}\right)\left(\frac{\delta}{r_2}\right)^3\frac{1}{(K')^2} \qquad (23)$$

To determine the conditions that must be met to allow replacing the cylindrical wave with a plane one when computing the power P (eq. (18)) the following ratios

$$\frac{P'}{P}=\frac{1}{\sqrt{2}p_1}\left(\frac{K}{K'}\right)^2, \qquad \frac{P''}{P}=\left(1-\frac{\delta}{2r_2}\right)\frac{P'}{P} \qquad (24)$$

have been calculated for various $\frac{r_2}{\delta}$ and $\frac{r_1}{r_2}$ on the assumption that $\mu_r=1$. The results of the calculations have been shown graphically in Figures 5 and 6. It can be seen from the plots that the power P'' (23) differs from the power P(18) to a lesser degree than the power P' (22), and the difference $P''-P$ depends on the values of the ratios $\frac{r_2}{\delta}$ and $\frac{r_1}{r_2}$. When $\frac{r_2}{\delta}>3$ and $\frac{r_1}{r_2}>1.2$, an approximate equality $P''\approx P$ occurs. Therefore, the cylindrical electromagnetic wave can be replaced with a plane one under these conditions when calculating the power P.

When assuming a plane electromagnetic wave, the calculated workpiece reactive power Q'' (23) equals the real power P''. A more accurate value of $Q''\approx Q$ (18) can be computed from equality $Q''=P''\frac{q_1}{p_1}$, where the value of the ratio $\frac{q_1}{p_1}$ can be read from the curve in Figure 2.

4. CONCLUSIONS

The magnetic flux induced in a long cylindrical and empty coil remains practically unchanged after putting a workpiece into the coil. The constancy of the flux makes it easier both to compute the amplitude of the magnetic field intensity H_{mo} at the surface of the workpiece heated by induction and to analyse various electric quantities in the workpiece, such as distributions of magnetic and electric field intensity and those of current and power density.

The analysis of the powers $P+jQ$ (18) in a workpiece of a fixed radius r_2 has shown that at a fixed coil voltage these powers fall rapidly both with increasing the ratio $\frac{r_2}{\delta}$, caused by a rise of coil current frequency, and by increasing coil radius r_1.

For loads with $\mu_r=1$ it is easy to determine the workpiece powers P and Q based on eqs. (20) and graphs in Figures 3 and 4. In case of $\mu_r>1$ the discussed powers P and Q can be computed using curves in Figure 1, the expression for K (10) and equations (19).

It has also been proved that in case of $\frac{r_2}{\delta}>3$ and $\frac{r_1}{r_2}>1.2$, the cylindrical electromagnetic wave can be replaced with a plane wave when calculating the workpiece real power P, because P'' (23) $\approx P$ (20).

It should be noted that the discussed method can be used when the lengths of the coil and the workpiece are at least 10 times their diameters. In that case the lines of the vector $\underline{B}$ (or $\underline{H}$) are practically parallel to each other except for short sections at the ends.

A simplification has been made in the analysis carried out above that the EMF induced in the coil with load approximately equals the voltage U supplying the coil. This approximation is the more accurate the lower the power factor of the coil with load. If $\cos\varphi = 0.1$ then $E = 0.995\,U$ and if $\cos\varphi = 0.2$ then $E = 0.98\,U$.

5. LIST OF SYMBOLS AND ABBREVIATIONS

A, B	— auxiliary quantities
E	— electromotive force (EMF) induced in the coil
E_m, E_m'	— electric field intensity, amplitude
f	— coil current frequency
h	— coil length
H_m, H_m'	— magnetic field intensity, amplitude
H_{mo}, H_{mo}'	— magnetic field intensity at the workpiece surface, amplitude
I, I_m	— coil current
j	— $\sqrt{-1}$ — imaginary unit
J_o, J_1	— Bessel functions
k	— $\dfrac{1+j}{\delta}$
K, K'	— auxiliary quantities
L	— coil inductance
p_1	— auxiliary quantity
p_{so}, p_{so}'	— surface density of workpiece real power
P, P', P''	— workpiece real power
q_1	— auxiliary quantity
q_{so}, q_{so}'	— surface density of workpiece reactive power
Q, Q', Q''	— workpiece reactive power
r	— distance measured from the workpiece centre along its radius
r_1	— coil internal radius
r_2	— workpiece radius
w	— number of coil turns
U	— supply voltage
x	— distance measured from the workpiece surface along its radius
γ	— workpiece electric conductivity
δ	— depth of penetration
μ	— workpiece magnetic permeability
μ_o	— magnetic permeability of free space
μ_r	— relative magnetic permeability
Φ_m	— magnetic flux, amplitude
Φ_{mp}	— magnetic flux in the air gap, amplitude
Φ_{mw}, Φ_{mw}'	— magnetic flux in the workpiece, amplitude
ω	— current angular frequency

6. REFERENCES

1. Davies E.J.: *Conduction and induction heating.* Peter Peregrinus Ltd., London, 1990.
2. Kurbiel A.: *Indukcyjne urządzenia elektrotermiczne* (*Induction Equipment for Electroheat*—in Polish), Wydawnictwa AGH. Kraków, 1992.
3. Langer E.: *Teorie indukčniho a dielektrického tepla* (*Theory of Induction and Dielectric Heating*—in Czech), Academia, Praha, 1979.
4. McLachlan, N.W.: *Bessel Functions for Engineers.* Oxford, At the Clarendon Press, 1955.
5. Sajdak C., Samek E.: *Nagrzewanie indukcyjne.* (*Induction heating*—in Polish) Wydawnictwo Śląsk, 1987.

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