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SURFACE EFFECTS IMPACT ON CONDUCTOR IMPEDANCE

Oddziaływanie zjawisk powierzchniowych na impedancję przewodnika

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Summary: **This article presents possibilities of practical application of numerical elementary conductors method for investigation of surface effects. Their influence on elements of conductor impedance result from of intensive electromagnetic fields action in the conductor surroundings. The method particuralry applies to calculation of skin effect and proximity effect coefficients in different shape conductors and configuration of bundle conductors.**

Streszczenie: W artykule przedstawiono możliwości praktycznego zastosowania numerycznej metody przewodników elementarnych do badania zjawisk powierzchniowych. Ich wpływ na elementy impedancji przewodnika jest wynikiem oddziaływania intensywnych pól elektromagnetycznych w otoczeniu przewodnika. Podana metoda znajduje zastosowanie szczególnie do obliczania współczynników zjawiska naskórkowości i zjawiska zbliżenia dla przewodników o różnych kształtach i dla różnych konfiguracji przewodów wiązkowych

Keywords: elementary conductors method, skineffect, proximity effect, conductor resistance and impedance, physical affinity

1. INTRODUCTION

Unevenly distributed current density on the conductor cross–section is not actual for current intensity about up to 2kA in electrotechnical experience. There is no problem to transfer electric power to the consumption place at these current values and right conductor dimensioning. Considering over 2kA current values electric power transfer becomes more difficult. In the conductor surround affect predominant electromagnetic fields that intensify surface effects on the larger cross – section so that these essentially influence conductor impedance and consequently their transfer ability.

Well-known example of objective task in the electrotechnical experience are so-called low–ohmic furnace supplies that is mechanism working in low voltage and high current mode. These are electrical furnaces with working impedance comparable to electric power supply impedance (glass melting furnace, arc furnace, electroslag eguipments, etc.).

In EPQU magazine, we have published preliminary article oriented to using numerical method possibility for counting of the conductor impedance components that are liable to alternate electromagnetic fields [1]. Especially we have presented elementary conductor method that come out from key idea of real conductor dividing to elementary ones and assumption if current density on the divided conductor cross – section is different then in each single elementary conductor will make constant value [2]. With reference to presented methodics of mathematical surface effects modelling with using elementary conductors, purpose of our report is to present some analysis results of electromagnetical field influence on supply impedance of simple and bundle conductor. In this first part confine to determinimg resistance change in supply conductor as a result of skineffect and proximity effect. Knowledge of these leads to optimalization of the cross–section, shape and supply conductor layuot in that way to insure power transfer with minimal electrical loss in intensive electromagnetic fields setting. This is one of the conditions of racionalization of electric power consumption by melting processes realized in low–ohmic furnaces.

2. PRACTICAL APPLICATION OF ELEMENTARY CONDUCTORS METHOD FOR IMPEDANCE CALCULATION

Conductor impedance self calculation using objective method is proceded by compiling a mathematical model. Proper model is shortly defined in article conclusion [1] as follows: It is an impedance calculation model for parallel conductors with different shape and cross–section using the method of dividing the real conductor to adequate number and shape of elementary conductors.

For rectangular cross–section conductors dividing to elementary ones with the same shape, conditions for dividing in axis x and y direction and dividing number n_x and n_y are kept, so as:

$$n_x \geq \frac{x_š}{a} \qquad n_y \geq \frac{y_v}{a} \tag{1}$$

where $x_š$ is a bar width, y_v is a bar height, "a" is a penetration depth of the electromagnetical field into the conductor. For sufficient conditions assurance (1) as well as calculating quality the actual conductor was at least four times divided in both co-ordinates directions. By more dividings accuracy is only a little advanced but calculation time is longer.

By dividing circle cross–section conductor are analogous dividing conditions defined as follows:

— in axis r direction with conductor radius R_v

$$n_r \geq 2\left(\frac{2\pi R_v}{a}\right) \tag{2}$$

— in axis j direction with outer radius R_v and inner conductor radius R_o

$$n_\varphi \geq 2\left(\frac{R-R_o}{a}\right) \tag{3}$$

For full–circle conductor is $R_o = 0$. Double dividing in conditions (2) and (3) is specific for calculating accuracy advance. Together at the circle conductor modelling elementary ring shape conductors were replaced by square ones dividing in axis r direction at least four times, in axis φ direction at least sixteen times (fig. 2.7, lit. [1]).

In both cases of real conductor cross–section, request for smaller sizes of elementary conductor in comparison with adequate penetration depth of the wave is a proper request for permanent and right calculation by using objective method.

A mathematical model in a form of programme for impedance calculating using elementary conductor method was created in a programme backround called MATLAB for WINDOWS, especially by reason of its adventageous working with matrices, complex numbers and equation system solution. MATLAB also includes procedures for matrix modification and matrix equation solution, that make entire programming easier. Algorithm of the task is relative to that [3].

3. USING OF PHYSICAL AFFINITY CRITERIA IN SURFACE EFFECTS ANALISIS

In gereral, surface effect represents unevenly distribution of current density on the cross – section of the examined object, conductor or bundle conductor, that is influenced by alternate electromagnetic field effects.

Considering an isolated conductor as an examined object (return conductor is not space-oriented), influence of self – alternative field in the objective conductor causes surface effect well known as skineffect. Considering optional conductor in a space–oriented bundle conductor as an examined object, influence of alternative fields of the rest conductors (except itself) on objective conductor is known as proximity effect. In both cases, the result of self resp. outside fields effect is resistance difference in conductor owerflowed by direct current (R_{dc}) and alternative current (R_{ac}). This reality is expressed by surface effect coefficient that is objective conductor resistances ratio, having same geometrical size and same material:

$$k_p = \frac{R_{ac}}{R_{dc}} \tag{4}$$

Surface effect coefficient is so that common expression for skineffect and proximity effect.

For i-th isolated conductor is identical with the skineffect coefficient:

$$k_{pi} = k_{si} = \frac{R_{ac}}{R_{dc}} \tag{5}$$

For the same i-th conductor in a bundle conductor of n-conductors is:

$$k_{pi} = k_{si} \cdot \prod_{j=1;\, j\neq i}^{n} k_{bij} \tag{6}$$

where k_{bij} is proximity effect coefficient between i-th and j-th bundle conductor.

Using any mathematical method, surface effect analysis is not an easy problem. It results from the fact, that a lot of variables, physical and geometrical parameters for the examined object are needed to be put into the mathematical model and these really make calculations more complicate and longer. Obtained results in absolute values refer only for particular shape and cross–section. This disadvantage is taken away by physical appearance laws excercised especially in experimental methods of physical programming. Application of effect and processes physical appearance in mathematical methods by indicators and appearance criteria allows to reduce variables number in the solved equations, on the other side to obtain results in a universal form, non–dimensional, so actual for all physical similar objects.

Physical modelling in electrotechnics offers an extended application and is published in many publications [4,5,6]. Surface effects modelling belongs to the area of electromagentical fields models. Composing of needed indicators and appearance criteria can be done using different methodics generally infered from model purpose and propriety of the equations describing particular effect, process or system. Considering that surface effects in the conductor are always related with unevenly distributed current density on its cross – section, Helmholtz equation is appropriate for making physical appearance conditions. For formulation of current density phasor for general environment it has a form:

$$\nabla^2 \ \mathbf{J} + k^2 \cdot \mathbf{J} = 0 \tag{7}$$

with complex constant of electromagnetical wave spread k. For particular conductive environment is:

$$\nabla^2 \mathbf{J} - j\omega\gamma\mu\, \mathbf{J} = 0 \tag{8}$$

considering conductivity γ and absolute permeability μ ($\omega = 2\pi f$).

If the penetration depth "a" is input to the equation (8) applicated to the conductor—original and conductor—model, then by using e.g. integral analogues method the primary appearance criteria is gained:

$$\pi_1 = \frac{L_M^2}{a_M^2} = \frac{L_O^2}{a_O^2} \tag{9}$$

with characteristic size of conductors L, where index "O" refers to the original and "M" refers to the model. By chosen appearance scales:

$$m_L = \frac{L_M}{L_O} \quad \text{and} \quad m_a = \frac{a_M}{a_O}$$

we get for criteria (9) proper appearance indicator:

$$\frac{m_L^2}{m_a^2} = 1 \quad \text{resp.} \quad m_L^2 = m_a^2 \tag{10}$$

Physical model (in abstract or real form) is more easily to make from the same material what in actual case means for material scales that $m_\gamma = m_\mu = 1$. Then, outgoing from mathematical expression of the penetration depth, the variable is only waving frequency, where appearance criteria π_1 can be replaced with more simple:

$$\pi_2 = L_M^2 \cdot f_M = L_O^2 \cdot f_O \tag{11}$$

Proper appearance indicator will be:

$$m_L^2 \cdot m_f = 1 \quad \text{resp.} \quad m_L^2 = \frac{1}{m_f} \tag{12}$$

By meeting of conditions (11) and (12) the field profile on the cross–section of original and model is analogical, so current density distribution is evenly. Therefore proper surface effect coeffient k_p on the original and model must be also equal. Tha scale of its appearance is:

$$m_{kp} = 1 \tag{13}$$

These rules were applied at mathematical modelling of surface effects and are also useble at skineffect a nd proximity effect evaluation.

For practical application of the elementary conductor method for praticular conductor resp. bundle conductor is criteria (11) modified of purpose to specify smallest number of parameters that exactly define the cross–sections and conductors layout. So another relative variables, criteria that make calculation easier and make results more general, are gained.

1.a. Criteria for cylinder cross–section single conductor

Because full–cylider conductor is exactly defined by radius r_{vonk} and hollow cylinder conductor by radius r_{vonk} and wall thickness h, in accordance with the condition (11) these criteria belong to them:

$$\pi_{2p} = f \cdot r_{vonk}^2 \quad \text{a} \quad \pi_{2d} = f \cdot h^2 \tag{14}$$

with physical consequence of attenuation electromagnetical wave intensity in the conductor. To obtain univerasl results in graphical expression it is appropriate to input another ones indicated characterictic scale of conductor, so appropriate appearance simplex. That is easily get form previous as:

$$\pi_3 = \sqrt{\frac{\pi_{2d}}{\pi_{2p}}} = \frac{h}{r_{vonk}} \tag{15}$$

it is used as parameter to define dependence of skineffect coefficient on attenuation electromagnetical wave intensity in the full or hollow cylinder conductor (π_{2p}).

1.b. Criteria for rectangular cross–section single conductor

Rectangular cross–section conductor is exactly defined by its thickness h and height b. So criteria (11) can be written in double form. As:

$$\pi_{2h} = f \cdot h^2 \quad \text{and} \quad \pi_{2b} = f \cdot b^2 \tag{16}$$

with the same phisical consequence as criteria (14). So appropriate appearance simplex (characterictic scale of conductor) will analogous with simplex (15):

$$\pi_3 = \sqrt{\frac{\pi_{2h}}{\pi_{2b}}} = \frac{h}{b} \tag{17}$$

2.a. Criteria for cylinder cross–section bundle conductor

Besides outer radius r_{vonk} and wall thickness of conductor bundle of n-conductors is dimensional defined also by axial clearences between single conductors. (Fig. 1)

Criteria π_{2p} a π_{2d} (14) resp. π_3 (15) keep actual . Another important appearance simplex is relative caracterictic scale indicated conductor clearence between single bundle conductors. Realising also criteria relavance

$$\pi_4 = f \cdot d^2 \tag{18}$$

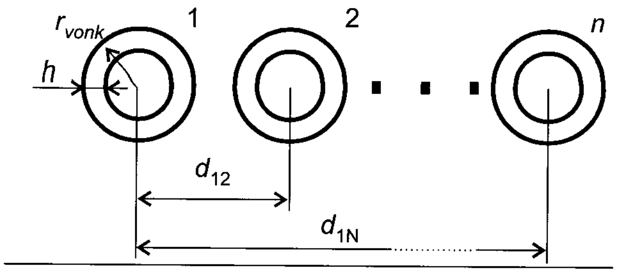

Fig. 1. Circle cross–section bundle conductor

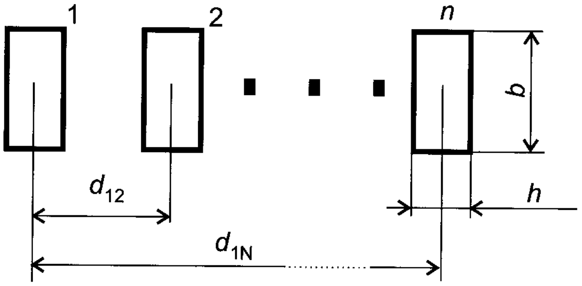

Fig. 2. Rectangular cross–section bundle conductor

search object is get easy from ratio:

$$\pi_5 = \sqrt{\frac{\pi_4}{\pi_{2p}}} = \frac{d}{r_{vonk}} \qquad (19)$$

Criteria π_{2p}, π_3 (15) and π_5 are sufficient for calculating and presenting results of proximity effect analisis of n-conductor bundle with cylinder cross–section.

2.b. Criteria for rectangular cross–section bundle conductor (Fig. 2)

With reference to single conductor (1.b.) criteria π_{2h}, π_{2b} (16) resp. π_3 (17) and with reference to n-conductor bundle (2a) criteria π_4 (18) stay actual. Additional criteria will be

$$\pi_6 = \sqrt{\frac{\pi_4}{\pi_{2h}}} = \frac{d}{h} \qquad (20)$$

All of them with the same concequence and application as it was stated in previous events.

In accordence with theoretical reasoning of the numerical elementary conductor method, presentation in the article [1] and together with previous notes, computing programs for surface effect solutions were made in the authors place of work. Some of the grafical results related to skineffect and proximity effect in the conductor are shown for demonstration. Values responding to copper that is electrical coductivity γ = 5,5.10^7$S.m^{-1}$ and relative permeability μ_r = 1 were chosen as material constants.

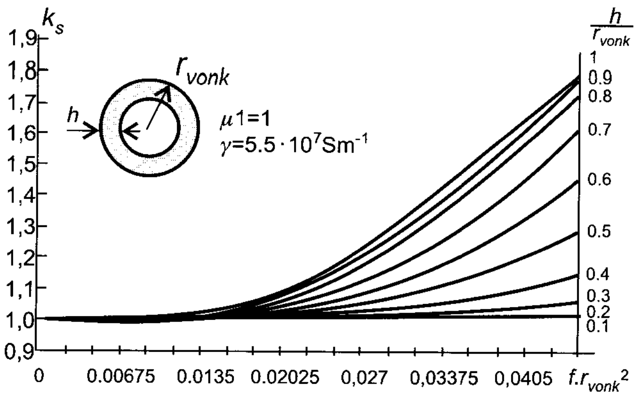

Fig. 3. Effect of criteria π_{2p} and π_3 on coefficient k_s for cylinder conductor

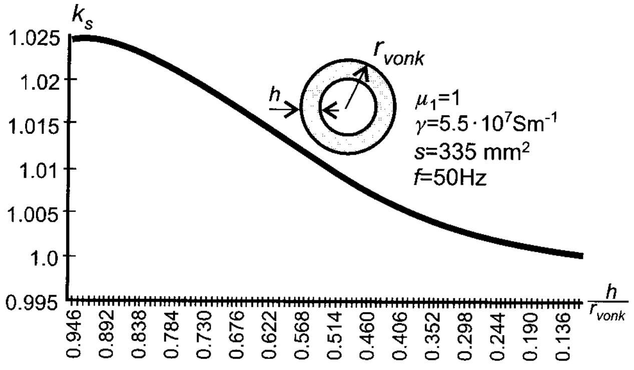

Fig. 4. Effect of wall thickness in the cylinder conductor on coefficient k_s

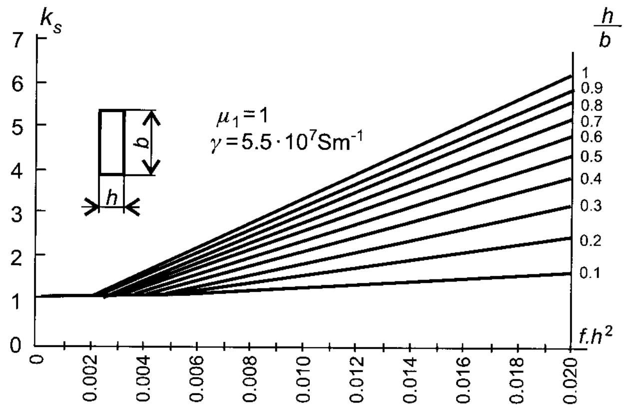

Fig. 5. Effect of criteria π_{2h} and π_3 on coefficient k_s for rectangular cross-section conductor

4. SOME SOLUTION RESULTS OF CONDUCTORS SKINEFFECT [3]

In the Fig. 3 there is shown the dependence of skineffect coefficient k_s of hollow cylinder cross–section on criteria π_{2p} (14) with curve parameter π_3 (15).

Result form curves behaviour is known physical fact that self-alternative field effect in the hollow cylinder conductor is toned up with increase of source frequence resp. conductor radius the more the larger relative depth of hollow conductor p$_3$. Skineffect is mostly expressed on the full–cylinder conductor ($h = r_{vonk}$). This fact is also supported in the Fig. 4 which expresses the function $k_s = \mathrm{f}(\pi_3)$ for particular conductor cross–section and source frequency.

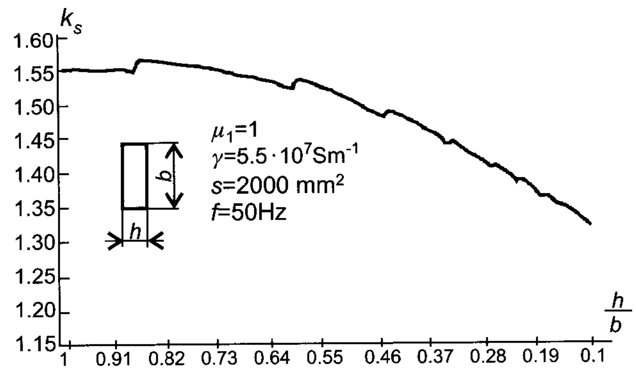

Fig. 6. Effect of the conductor shape with rectangular cross–section on the coefficient k_s

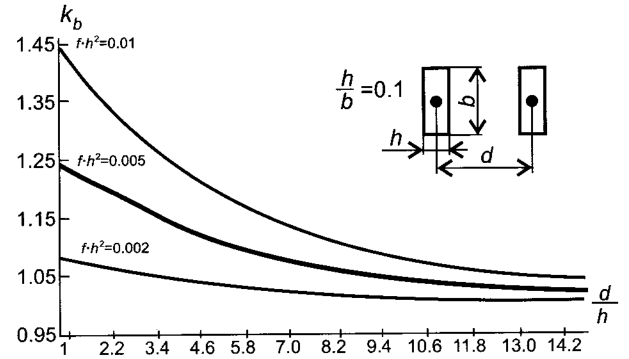

Fig. 7. Relative distance effect between strips on k_b

Shown curve also expresses change of conductor resistance R_{ac} because for chosen cross–section is R_{dc} = const. Chosen cross–section 335 mm^2 responds to hollow copper conductor, cooled by water, with permitted current density 3A.mm^{-2} by current—carrying capacity 1kA.

Analogous dependences were confirmed also for conductor with rectangular cross–section shape. Fig. 5 shows the function $k_s = f(\pi_{2h})$ with parameter π_3 (17).

Also in this case, analogous to cylinder conductor, coefficient k_s insreases with more intensive self field in the conductor as well as little by little change of its cross–section form rectange shaped to square shaped (Fig. 6).

Unevenly curve is not caused by physical anomaly of examined effect, but it is caused by gross dividing of conductor to elementary ones. Conductor with rectangular cross–section 2 000 mm^2 refers to permitted current density 1,5A.mm^2 by current–carrying capacity 3kA.

5. SOME RESULTS FOR PROXIMITY EFFECT SOLUTION IN BUNDLE CONDUCTOR

Next Figure 7 expresses dependency of $k_b = f(\pi_6)$ with parameter π_{2h} for the same cross-section 2000 mm^2 of each conductor pair. The value of coefficient k_b with increasing relative distance between strips of course decrease, because field effect of adjacent conductor decreases.

Fig. 8 provides an interesting results from the view of shape and location of strip conductors in the space. It demonstrates dependency of $k_b = f(\pi_3)$ that is change of coefficient k_b from rectangular conductor shape.

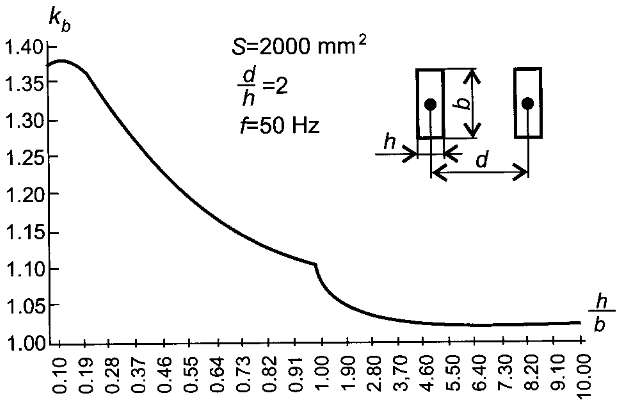

Fig. 8. Effect of cross-section shape of bundle conductors on coefficient k_b

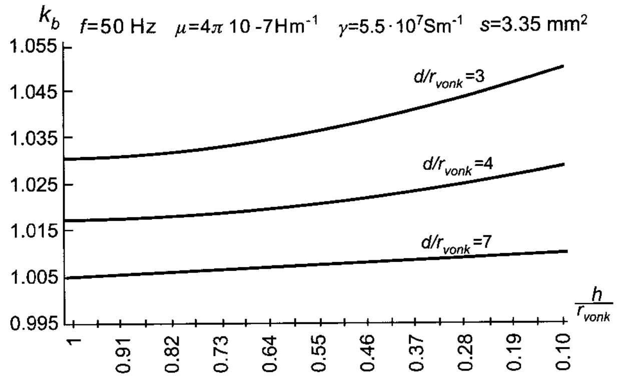

Fig. 9. Effect of shape and distance of circular cross-section conductors on coefficient k_b

Matter of result interest is that electric energy transfer is most energy convenient with strip conductors located horizonally in space.

At least in the Fig. 9 there is shown function $k_b = f(\pi_3)$ with parameter π_5. It expresses effect of given criteria on coefficient k_b size of two circular cross-section conductors.

6. CONCLUSION

Presented examples of skineffect and proximity effect solution are not comprehensive for usage of elementary conductors method. Except of sufficient precision, adventage of the method is mainly in analysis providing of proximity effect resp. total surface effect also on more difficult bundle shape conductors. For example it is two-conductor bundle with same conductor shape or bundle of larger number of conductors and different relative configuration in space. These all are real examples of supply circuit conductors layout in electric furnaces working in low-ohmic mode. With respect to systematic endeavor to decrease electric losses in their operation, application of elementary conductor method for surface effects analysis can be considered as specially actual at present possibilities of computer art.

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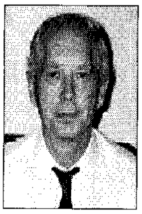

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