DYNAMIC STATES OF THE ELECTRIC ARC IN CONTROLLED CURRENT SUPPLY SYSTEMS OF ELECTROTHERMAL DIRECT CURRENT DEVICES

Part 2. General Stability Conditions of Electric Arc Dynamic Systems

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Summary: **A new interpretation of Kaufmann criterion has been described in the paper. The criterion concerns a stability of electrical D.C. arc in the simple electrical circuit with an additional resistance. A chemical composition of plasmagenic gas and three methods of dissipation of energy arc in the form of conduction, radiation and the convection head are considered in the paper. In the case of a rectifier thyrystor system supply of arc, the transfer function of linearized system is described. The criterion of minimal inductance choke needed for stabilization of arc in the electrical circuit is shown.**

1. INTRODUCTION

The modelling of the electric arc by means of non-linear voltage-current characteristics leads to the classical Kaufmann stability criterion. The criterion does not, however, provide the relation of the parameters and arc power characteristics to the required resistance in the electric circuit. The application of the two-layer arc model makes it possible to specify the criterion by relating it to the thermal and electrical characteristics of plasma within the arc column. The analysis of dynamic states in controlled current supply systems of industrial plasmatrons is complicated by the presence of thyristor converters, which are significantly non-linear components with low ability of being controlled. The locally linearized differential equation and the transmittance of the direct current arc described in Part 1 [1] are the input data for developing new stability criteria and for the selection of a choke in the power circuit.

2. THERMOPHYSICAL INTERPRETATION OF THE KAUFMANN CRITERION

The classical problem of arc stability concerns the simplest circuit with resistor R and non-controlled current supply ($u_s(\tau) = U_s =$ const) described by the following:

$$U_s = u_a(\tau) + i \cdot R \tag{1}$$

where U_s is the voltage at the current supply; u_a is the arc voltage; R is the additional resistance.
In terms of relative increments Eqn. (1) is:

$$\Delta\bar{u}_a = -\frac{R \cdot g_c(i_0)}{l_a} \cdot \Delta\bar{i} \tag{2}$$

where $\Delta\bar{u}_a = \dfrac{\Delta u_a}{U_c}$ is relative deviation of the voltage component, U_c is the component of the arc voltage in the equilibrium point of the dynamic system, corresponding to current $I = i_0$, l_a is the arc length, g_c is the conductance of the cylindrical part of a 1-meter-long arc. After substituting (2) into Eqn. (66) [1] we obtain:

$$\Theta_a \cdot \left(1 + \frac{R \cdot g_c(i_0)}{l_a}\right) \cdot \frac{\mathrm{d}\Delta \bar{i}}{\mathrm{d}\tau} + \\ + \left(1 - k_a + \frac{R \cdot g_c(i_0)}{l_a}\right) \cdot \Delta \bar{i} = 0 \tag{3}$$

where Θ_a is the time-constant of electrical arc.

According to (3) the thermophysical conditions of arc stability have the following form:

$$\Theta_a > 0\,; \quad \left[\frac{\mathrm{d}h_c(g_c)}{\mathrm{d}g_c}\right]_{i_0} > 0 \tag{4}$$

$$R > \frac{(1-k_a) \cdot l_a}{g_c(i_0)} = \frac{(1-k_c+k_k) \cdot l_a}{g_c(i_0)} \tag{5}$$

After substituting factors k_c (55) and k_k (42) [1] into (5) we arrive at:

$$R > \frac{\left\{1 - \frac{2i_0^2}{p_c(i_0) \cdot g_c(i_0) + \left[\frac{\mathrm{d}p_c(g_c)}{\mathrm{d}g_c}\right]_{i_0} \cdot g_c^2(i_0)} + \left[\frac{\mathrm{d}u_{kon}(i)}{\mathrm{d}i}\right]_{i_0} \cdot \frac{g_c(i_0)}{l_a}\right\} \cdot l_a}{g_c(i_0)} \tag{6}$$

From Eqn. (66) [1] one can obtain the formula for arc dynamic resistance ${}_dR_a$. Let us rewrite this equation for the equilibrium point $[i_0, g_c(i_0)]$ of the dynamic arc:

$$\Delta \bar{u}_a = (1 - k_a) \cdot \Delta \bar{i} \tag{7}$$

After transforming Eqn. (7) we obtain the following in absolute increments:

$${}_dR_a = \frac{\Delta u_a}{\Delta i} = \frac{(1-k_a) \cdot l_a}{g_c(i_0)} = \frac{(1-k_c+k_k) \cdot l_a}{g_c(i_0)} \tag{8}$$

Comparing (8) and (5) leads to an important conclusion. The condition of arc stability (6) constitutes a thermophysical interpretation of the classical Kaufmann criterion:

$$R > -{}_dR_a = -\left(\frac{\mathrm{d}U_a(I)}{\mathrm{d}I}\right)_{i_0} = -\left(\frac{\Delta u_a}{\Delta i}\right)_{i_0} \tag{9}$$

where $U_a(I)$ is steady—state arc characteristic.

3. CALCULATING ARC STABILITY IN CONTROLLED CURRENT SUPPLY SYSTEMS OF ARC AND PLASMA DC DEVICES

The classical criterion of arc stability (9) indicates only that if the arc external characteristic is steep, then the characteristic at the feeding source of the device must also be steep. It is necessary, but not a sufficient condition for designing controlled thyristor systems, which are commonly applied in electrotechnological arc and plasma devices [2, 3]. A typical schema of a feeding source in a high power plasma device is shown in Fig. 1. The source contains an automatic arc current controller ensuring the steep shape of the characteristic (Fig. 2). The thyristor current supply of the arc load has a limited ability of controlling the dynamic state of the arc due to the commensurability of the relaxation time of the arc electric state and the discretization interval of the thyristor converter. Because of that, it is necessary to use smoothing impedance coil L in every thyristor current supply system in order to achieve a stable arc. The coil is connected in series with the arc in the load circuit (Fig. 1). A fundamental problem of designing current supply systems of plasma and arc devices is calculating the value of inductance L, which would secure the stability of arc burning in the circuit with a selected steepness of the external characteristic of the source. The steepness is defined by reinforcement coefficient k_s of the arc current controller. The developed model of the dynamic arc makes it also possible to estimate the thermophysical condition of arc stability (4).

In order to research the stability of the circuit analytically, first-degree simplification assumption is considered. The phase-controlled rectifier is replaced by an "uncontrollable" rectifier with voltage amplitude modulation [4]. The discretization intervals are equal and their boundaries refer to natural commutation points. A smooth component of rectified voltage is obtained at the output of the device. Because the dynamic properties of arc depend on enthalpy function $h_c(g_c)$ and on dissipated power $p_c(g_c)$ of the arc electric field, for the purpose of modelling the thyristor converter it is sufficient to analyse the dynamic parameters in the form of mean value ${}_k\hat{y}_{me}$ of instantaneous functions at each discretization interval of the thyristor converter [4]

$$y(\tau) = \sum_{k=-\infty}^{k=+\infty} {}_k\hat{y}_{me} \tag{10}$$

If the instantaneous parameters of the thyristor system are being modelled, this condition prohibits passing from phase to amplitude modulation (Fig. 3)

$$y(\tau) = \frac{1}{T_k} \int_0^{T_k} \sum_{k=n-1}^{k=n} y_{me} \cdot \mathrm{d}\tau \tag{11}$$

where k is the number of digitization interval, $n-1$, n are the boundaries of smooth function y_{me} digitization intervals.

The influence of transient processes caused by commutation of valves will be neglected. We shall present a mathematical model of the thyristor converter consisting of the following (Fig. 4):

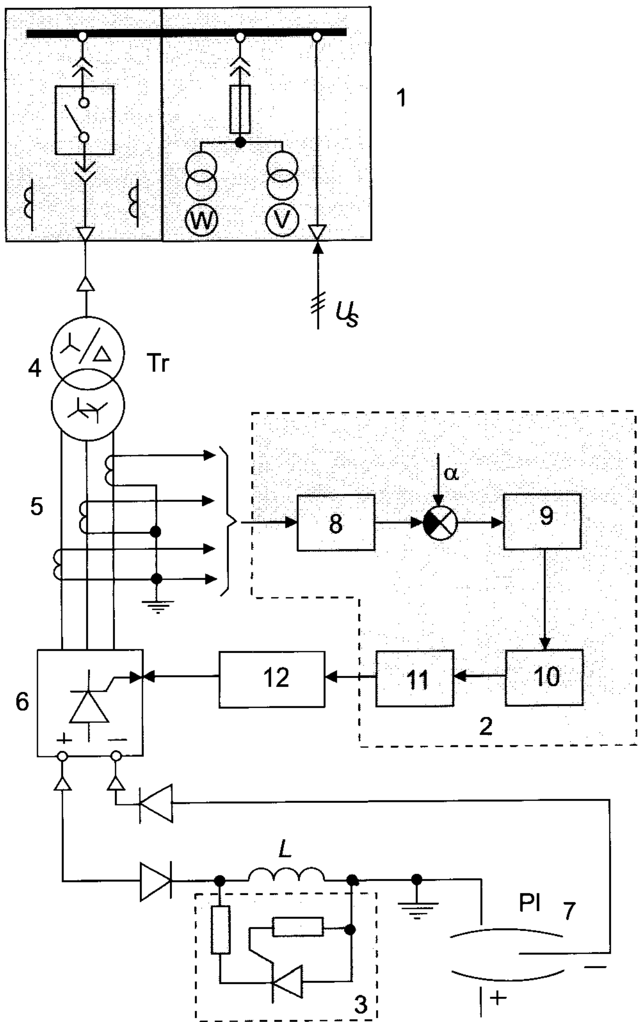

Fig. 1. Scheme of a Controlled Current Supply System of Industrial Plasmatrons (1—Switching Station; 2—Automatic Controller of Arc Current; 3—Protection System; 4—Transformer; 5—Current Transformer; 6—Thyristor Rectifier; 7—Plasmatron; 8—Measuring Block; 9—Control Forming System; 10—Pulse Forming Block; 11—Power Amplifier; 12—Phase-Pulse Control System)

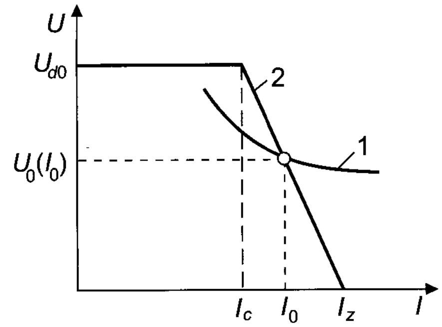

Fig. 2. Linearized External Static Characteristic of the Controlled Rectifier for Plasmatron Current Supply (1—External Characteristic of Arc; 2—External Characteristic of Current Supply Source)

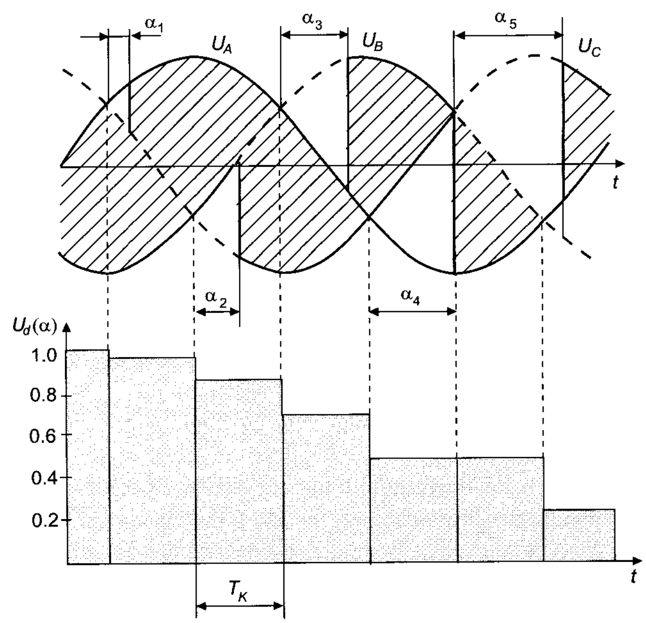

Fig. 3. Passing from Phase Modulation to Pulse Modulation of the Signal at the Output of the Controlled Rectifier

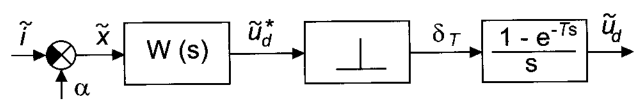

Fig. 4. Mathematical Model of Locally Linearized Controlled Rectifier (Tilde Above a Quantity Denotes Laplace's Transformation of Relative Function; $\tilde{u}_d^*$ —Continuous Signal of Thyristor Converter Voltage, δ_T —Comb-shaped Function of the Pulse System; $\tilde{u}_d$ —Output Pulse Signal of the Converter)

a) continuous part with transmittance $W_s(s)$ of the current controller;
b) pulse component;
c) extrapolator of zero order.

In most arc and plasma devices current controllers P or PI are used. The type of the controller does not influence the arc stability. However, it influences the stability of the whole circuit with arc. Because of that it suffices to calculate the arc stability in a dynamic system with true-life controller P with inertia. In such a case transmittance $W_s(s)$ of the controller is in the typical form of first order inertial branch:

$$W_s(s) = \frac{k_s}{\Theta_s \cdot s + 1} \qquad (12)$$

where Θ_s results from discrete—type mode of rectifier's operation. Reinforcement coefficient k_s of the controller determines the degree of steepness of the external characteristic of the thyristor converter:

$$k_s = \left(\frac{dU_d(I)}{dI}\right)_{i_0} \cdot G_a(i_0) \qquad (13)$$

where $U_d(I)$ is steady–state power supply characteristic; $G_a(i_0)$ is the conductance of arc in the point of the dynamic system with current i_0.

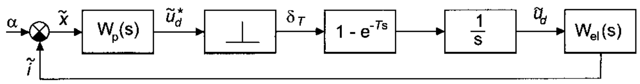

Fig. 5. Structural Scheme of the Controlled Current Supply System of a D.C. Arc ($\tilde{i}$ —Signal of Current in Arc Circuit)

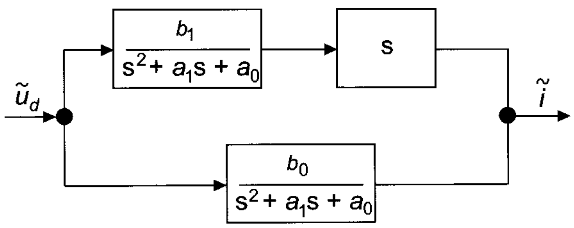

Fig. 6. Structural Scheme of Circuit *RL* with an Arc

In the practice of designing and exploitation [3] of controlled sources of current supply in plasma devices the following coefficient k_s of the current controller is assumed:

$$k_s = -(3 \div 8) \tag{14}$$

To ensure economical operating conditions of the thyristor current supply it is necessary to select time constant Θ_s of the current controller. It should not exceed the discreteness interval of the thyristor converter [3]:

$$\Theta_s \leq T \tag{15}$$

The regime of economical work requires minimal value of inductance of stabilizing choke in the arc circuit and minimum reserves choke of transformer's secondary—coil and rectified voltages. If the controller's inertia Θ_s increases, one ought to decrease k_s. Then dU/dI decreases and the current control error increases. The increase of controller's inertia Θ_s requires the increase of choke's inductance L. If rectifier voltage reserve increases, the power coefficient cosφ decreases and harmful interaction with the electric grid occurs. Now let us construct a mathematical model of current supply source of D.C. arc, which will be used for calculating inductance L of the smoothing choke. Fig. 5 shows the structural scheme of the current supply source. It contains a model of the thyristor converter with the arc current controller. In this model transmittance $W_{el}(s)$ describes the dynamics of the rectified current circuit of the converter. In the general form the circuit containing the arc and the impedance coil L has some resistance R. Let us write the circuit equation:

$$\Delta\bar{u}_a = \Delta\bar{u}_d - R \cdot G_a(i_0) \cdot \Delta\bar{i} - L \cdot G_a(i_0) \cdot \frac{d\Delta\bar{i}}{d\tau} \tag{16}$$

which we will use for deriving transmittance $W_{el}(s)$. Relative deviation of rectifier voltage is denoted $\Delta\bar{u}_d = \frac{\Delta U_d}{U_c}$.

The arc model is simplified by neglecting of conical part. If we assume that inertia indicators are equal $\Theta_c = \Theta_a$, then we can obtain an analytic solution of the stability problem of the thyristor system. In this case Eqn. (66) [1] takes the following, simpler form:

$$\Theta_a \cdot \frac{d\Delta\bar{u}_a}{d\tau} + \Delta\bar{u}_a = \Theta_a \cdot \frac{d\Delta\bar{i}}{d\tau} + (1 - k_a) \cdot \Delta\bar{i} \tag{17}$$

After we substitute (16) into the above we will obtain the equation of the load circuit of the thyristor source (Fig 6):

$$\frac{d^2\Delta\bar{i}}{d\tau^2} + a_1 \cdot \frac{d\Delta\bar{i}}{d\tau} + a_0 \cdot \Delta\bar{i} = b_1 \cdot \frac{d\Delta\bar{u}_d}{d\tau} + b_0 \cdot \Delta\bar{u}_d \tag{18}$$

The coefficients of this equation are:

$$a_0 = \frac{R \cdot G_a(i_0) + 1 - k_a}{\Theta_a \cdot L \cdot G_a(i_0)}$$

$$a_1 = \frac{\Theta_a + \Theta_a \cdot R \cdot G_a(i_0) + L \cdot G_a(i_0)}{\Theta_a \cdot L \cdot G_a(i_0)} \tag{19}$$

$$b_0 = \frac{1}{\Theta_a \cdot L \cdot G_a(i_0)}; \qquad b_1 = \frac{1}{L \cdot G_a(i_0)} \tag{20}$$

According to Eqn. (18) the transmittance of the rectified current arc circuit is (Fig. 6):

$$W_{el}(s) = \frac{b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0} \tag{21}$$

In this way, the mathematical model of the dynamic system with an arc and controlled current supply source can be represented by a structural scheme of a pulse system (Fig. 5) containing a pulse component, a zero-order extrapolator and with transmittances $W_s(s)$ and $W_{el}(s)$ of the arc current controller (12) and of the load circuit (21), respectively. The transmittance of this system confined to plane z [3] is the following:

$$W(z,0) = \frac{\alpha_4 \cdot z^4 + \alpha_3 \cdot z^3 + \alpha_2 \cdot z^2 + \alpha_1 \cdot z}{\beta_4 \cdot z^4 + \beta_3 \cdot z^3 + \beta_2 \cdot z^2 + \beta_1 \cdot z^1 + \beta_0} \tag{22}$$

The transmittance coefficients have the following general form:

$$
\begin{aligned}
\alpha_1 = {} & \frac{k_s \cdot \Theta_a}{\Theta_a + L \cdot g_a(i_0)} \cdot \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot g_a(i_0))}{\Theta_a \cdot L \cdot g_a(i_0)} + \frac{T}{\Theta_a} \right] \right\} + \\
& + \frac{k_s \cdot L \cdot g_a(i_0) \cdot (\Theta_a + \Theta_s) \cdot \Theta_a \cdot \exp\left(-\frac{T}{\Theta_s} \right)}{(\Theta_a + L \cdot g_a(i_0)) \cdot [(\Theta_a + L \cdot g_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot g_a(i_0)]} - \\
& - \frac{k_s \cdot L \cdot g_a(i_0) \cdot (\Theta_a + 2L \cdot g_a(i_0)) \cdot \Theta_a \cdot \exp\left(-\frac{T}{\Theta_s} \right)}{(\Theta_a + L \cdot g_a(i_0)) \cdot [(\Theta_a + L \cdot g_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot g_a(i_0)]} - \\
& - \frac{k_s}{\Theta_a + L \cdot g_a(i_0)} \cdot \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot g_a(i_0))}{\Theta_a \cdot L \cdot g_a(i_0)} + \frac{T}{\Theta_s} \right] \right\}
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
\alpha_2 = {} & \frac{k_s \cdot \Theta_a}{\Theta_a + L \cdot g_a(i_0)} \cdot \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot g_a(i_0))}{\Theta_a \cdot L \cdot g_a(i_0)} \right] \right\} - \\
& - \frac{k_s \cdot T}{\Theta_a + L \cdot g_a(i_0)} + \frac{k_s \cdot \Theta_a}{\Theta_a + L \cdot g_a(i_0)} \cdot \\
& \cdot \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot g_a(i_0))}{\Theta_a \cdot L \cdot g_a(i_0)} + \frac{T}{\Theta_s} \right] \right\} + \\
& + \frac{k_s \cdot \Theta_a \cdot [L \cdot g_a(i_0) \cdot (\Theta_a + \Theta_s) - \Theta_s \cdot (\Theta_a + 2L \cdot g_a(i_0))]}{(\Theta_a + L \cdot g_a(i_0)) \cdot [(\Theta_a + L \cdot g_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot g_a(i_0)]} \cdot \\
& \cdot \left\{ 1 + 2 \cdot \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot g_a(i_0))}{\Theta_a \cdot L \cdot g_a(i_0)} \right] \right\} \right\}
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
\alpha_3 = {} & \frac{k_s \cdot T}{\Theta_a + L \cdot g_a(i_0)} - \frac{k_s \cdot \Theta_a}{\Theta_a + L \cdot g_a(i_0)} \cdot \\
& \cdot \left\langle 1 + \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot g_a(i_0))}{\Theta_a \cdot L \cdot g_a(i_0)} \right] \right\} + \exp\left(-\frac{T}{\Theta_s} \right) \right\rangle - \\
& - \frac{k_s \cdot \Theta_a \cdot [L \cdot g_a(i_0) \cdot (\Theta_a + \Theta_s) - \Theta_s \cdot (\Theta_a + 2L \cdot g_a(i_0))]}{(\Theta_a + L \cdot g_a(i_0)) \cdot [(\Theta_a + L \cdot g_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot g_a(i_0)]} \cdot \\
& \cdot \left\{ 2 + \exp\left[-\frac{T}{\Theta_s} \right] \right\}
\end{aligned} \tag{25}
$$

$$
\alpha_4 = \frac{k_s \cdot \Theta_a \cdot [L \cdot g_a(i_0) \cdot (\Theta_a + \Theta_s) - \Theta_s \cdot (\Theta_a + 2L \cdot g_a(i_0))]}{(\Theta_a + L \cdot g_a(i_0)) \cdot [(\Theta_a + L \cdot g_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot g_a(i_0)]} \tag{26}
$$

$$
\beta_0 = \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} + \frac{T}{\Theta_s} \right] \right\} \tag{27}
$$

$$
\begin{aligned}
\beta_1 = {} & \alpha_1 - 2 \cdot \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} + \frac{T}{\Theta_s} \right] \right\} + \\
& + \exp\left[-\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] + \exp\left(-\frac{T}{\Theta_s} \right)
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
\beta_2 = {} & \alpha_2 + 1 - \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} + \frac{T}{\Theta_s} \right] \right\} \\
& - 2 \cdot \exp\left[-\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] - 2 \cdot \exp\left(-\frac{T}{\Theta_s} \right)
\end{aligned} \tag{29}
$$

$$
\beta_3 = \alpha_3 + 2 + \exp\left[-\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] + \exp\left(-\frac{T}{\Theta_s} \right) \tag{30}
$$

$$
\beta_4 = \alpha_4 + 1 \tag{31}
$$

Let us now introduce a new variable according to the bilinear transformation method:

$$
z = \frac{1 + w}{1 - w} \tag{32}
$$

In this case the characteristic equation (21) of the system becomes:

$$
\begin{aligned}
& \beta_4 \cdot \left(\frac{1+w}{1-w} \right)^4 + \beta_3 \cdot \left(\frac{1+w}{1-w} \right)^3 + \\
& + \beta_2 \cdot \left(\frac{1+w}{1-w} \right)^2 + \beta_1 \cdot \left(\frac{1+w}{1-w} \right) + \beta_0 = 0
\end{aligned} \tag{33}
$$

After performing the necessary algebraic transformations we arrive at the characteristic equation of the current supply system. As a result of representing the left-hand side on complex plane z we have:

$$\gamma_4 \cdot w^4 + \gamma_3 \cdot w^3 + \gamma_2 \cdot w^2 + \gamma_1 \cdot w + \gamma_0 = 0 \qquad (34)$$

The coefficients of this equation are the following:

$$\gamma_0 = \beta_4 + \beta_3 + \beta_2 + \beta_1 + \beta_0; \; \gamma_1 = 4 \cdot \beta_4 + 3 \cdot \beta_3 + 2 \cdot \beta_2 + \beta_1;$$

$$\gamma_2 = 6 \cdot \beta_4 + 3 \cdot \beta_3 + \beta_2; \; \gamma_3 = 4 \cdot \beta_4 + \beta_3; \; \gamma_4 = \beta_4; \qquad (35)$$

According to the Hurwitz criterion the condition of arc stability in the thyristor converter system has the following form:

$$\gamma_0 = \beta_4 + \beta_3 + \beta_2 + \beta_1 + \beta_0 > 0$$

$$\gamma_3 = 4 \cdot \beta_4 + \beta_3 > 0$$

$$\gamma_3 \cdot \gamma_2 \cdot \gamma_1 - \gamma_3^2 \cdot \gamma_0 - \gamma_1^2 \cdot \gamma_4 > 0 \qquad (36)$$

$$\gamma_3 \cdot \gamma_2 - \gamma_1 \cdot \gamma_4 > 0$$

For the purpose of designing current supply systems for arc and plasma devices, the first two arc stability criteria are more strict and interesting (36). After substituting the parameters of the dynamic system into the first condition, we arrive at the general form:

$$\frac{2 \cdot k_s \cdot \Theta_a \cdot \left\{ \exp\left[-\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] - \exp\left(-\frac{T}{\Theta_s} \right) \right\}}{(\Theta_a + L \cdot G_a(i_0)) \cdot [(\Theta_a + L \cdot G_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot G_a(i_0)]} \times$$

$$\times [(\Theta_a + \Theta_s) \cdot L \cdot G_a(i_0) - (\Theta_a + 2 \cdot L \cdot G_a(i_0)) \cdot \Theta_s]$$

$$- \frac{k_s \cdot \Theta_a}{\Theta_a + L \cdot G_a(i_0)} + \frac{k_s \cdot T}{\Theta_a + L \cdot G_a(i_0)} \exp\left(-\frac{T}{\Theta_s} \right) \times$$

$$\times \left\{ \exp\left[-\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] - 1 \right\} > \qquad (37)$$

$$2 \cdot \left\{ 1 + \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} + \frac{T}{\Theta_s} \right] \right\} - \exp\left(-\frac{T}{\Theta_s} \right) - \right.$$

$$\left. - \exp\left\{ -\left[\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] \right\} \right\}$$

The second arc stability condition in the controlled current supply system is:

$$\frac{2k_s \cdot \Theta_a \cdot [(\Theta_a + \Theta_s) \cdot L \cdot G_a(i_0) - (\Theta_a + 2L \cdot G_a(i_0)) \cdot \Theta_s]}{(\Theta_a + L \cdot G_a(i_0))[(\Theta_a + 2L \cdot G_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot G_a(i_0)]} +$$

$$+ \frac{4k_s \cdot \Theta_a \cdot \left[2 + \exp\left(-\frac{T}{\Theta_s} \right) \right] \cdot [(\Theta_a + 2L \cdot G_a(i_0)) \cdot \Theta_s - L \cdot G_a(i_0) \cdot (\Theta_s + \Theta_a)]}{(\Theta_a + L \cdot G_a(i_0))[(\Theta_a + 2L \cdot G_a(i_0)) \cdot \Theta_s - \Theta_a \cdot L \cdot G_a(i_0)]} +$$

$$+ \frac{k_s}{\Theta_a + L \cdot G_a(i_0)} \cdot \left\{ T - \Theta_a \cdot \left\{ 1 + \exp\left[-\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] + \exp\left(-\frac{T}{\Theta_s} \right) \right\} \right\} +$$

$$+ 2 + \exp\left[-\frac{T \cdot (\Theta_a + L \cdot G_a(i_0))}{\Theta_a \cdot L \cdot G_a(i_0)} \right] + \exp\left(-\frac{T}{\Theta_s} \right) > 0 \qquad (38)$$

In designing thyristor current supply systems of arc and plasma devices we have to assume:

$$L \cdot G_a(i_0) >> \Theta_a \qquad (39)$$

In this case the first arc stability condition (36) takes the simpler form:

$$\frac{L \cdot G_a(i_0)}{\Theta_a} > \frac{k_s}{2} \cdot \frac{\left(2 - \frac{T}{\Theta_a} \right) \cdot \exp\left(-\frac{T}{\Theta_s} \right) - 1}{1 + \exp\left[-\left(\frac{T}{\Theta_a} + \frac{T}{\Theta_s} \right) \right] - \exp\left(-\frac{T}{\Theta_s} \right) - \exp\left(-\frac{T}{\Theta_a} \right)} \qquad (40)$$

The second arc stability condition (38) can be described by the following inequality:

$$\frac{L \cdot G_a(i_0)}{\Theta_a} > \frac{k_s \cdot \left[3 - \exp\left(-\frac{T}{\Theta_a} \right) - \frac{T}{\Theta_a} \right]}{6 + \exp\left(-\frac{T}{\Theta_s} \right) + \exp\left(-\frac{T}{\Theta_a} \right)} \qquad (41)$$

The comparison of the two inequalities indicates that the arc stability criterion (41) is rigid as compared to condition (40). Thus, in the calculations of the required inductance of smoothing impedance coil L, condition (40) can be used

$$L > \frac{\Theta_a}{2} \cdot \left(\frac{\mathrm{d}U_d(I)}{\mathrm{d}I} \right)_{i_0} \cdot$$

$$\cdot \frac{\left(2 - \frac{T}{\Theta_a}\right) \cdot \exp\left(-\frac{T}{\Theta_s}\right) - 1}{1 + \exp\left[-\left(\frac{T}{\Theta_a} + \frac{T}{\Theta_s}\right)\right] - \exp\left(-\frac{T}{\Theta_s}\right) - \exp\left(-\frac{T}{\Theta_a}\right)} \qquad (42)$$

4. CONCLUSIONS

The present analysis uses the two-layer, heterogeneous arc model for specifying stability conditions of arc burning in arc and plasma devices. In order to calculate the parameters of a designed current supply source one has to know an external electric characteristic of D.C. arc (e.g. static voltage-current characteristic) and thermal characteristics (of enthalpy, of linear density of power dissipated by conduction and by plasma radiation). The characteristics depend on the conditions of heat exchange in a specific device. The developed arc stability criteria make it possible to select resistance in a non-controlled feeding circuit and inductance in a system with a thyristor rectifier.

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