DYNAMIC STATES OF THE ELECTRIC ARC IN CONTROLLED CURRENT SUPPLY SYSTEMS OF ELECTROTHERMAL DIRECT CURRENT DEVICES

Part 1. Transmittance of the Electric Arc

Anatoli M. Krouchinin **Antoni Sawicki**
Technical University of Częstochowa
Poland

Summary: **In this article a solution is given of differential equation representing the energy of the cylindrical part of an unstationary arc. Universal functions of conductance, radiation and enthalpy are introduced on the basis of the physical model of two-layer arc. Similarity conditions of arcs in gas of the same chemical composition are specified. The influence of the conical part and of external disturbances on the model of dynamic arc is analysed. The transmittance of the arc has been obtained by local linearization of its characteristics. The relations holding between the arc transmittance parameters (time constant and coefficients) and the parameters of plasmagenic gas and of heat exchange coefficients are also given.**

1. INTRODUCTION

Increasing technological demands and the necessity to save electric energy call for new methods of ensuring the stability of arc burning in electrotechnological devices, especially in plasma devices, in which the extinction of arc is a very serious technological problem and it is considered to be an emergency state. The first studies of the dynamic properties and of the stability of arc burning referred to the processes of exstinguishing the arc. The obtained criteria of the local stability of a linking arc in electric devices were both necessary and sufficient, e.g. Kaufmann and Myer's criteria [1]. However, the only result of these studies is a qualitative analysis of dynamic non-linear characteristics of unstationary arc. Specifically, the authors of those considerations discovered the dependence of the time constant of the unstationary linearized arc on its conductance and current. But in the calculations of arc circuits, the nonlinear arc characteristics are neglected by using various methods of linearization of the energy balance differential equation. This makes it difficult to use them for the calculation of electric arc devices. In publications on the global arc stability the methods of qualitative theory of differential equations are applied, but, unfortunately, they are based upon the linear model of arc. A number of experiments has led to the mathematical model of unstationary two-layer arc [2]. It was applicable only to the cylindrical part of the column but not to the processes in the conical part and the areas near the electrodes. In the present cycle of papers the processes of heat exchange in those areas will be taken into consideration. The developed arc mathematical models also make it possible to take into account the chemical composition of plasmagenic gas and the construction of a device. Because of that it is possible to select the parameters of the source and the whole feeding circuit in such a way, that the arc discharge is stable.

2. THE SOLUTION OF DIFFERENTIAL EQUATION REPRESENTING THE ENERGY OF THE CYLINDRICAL PART OF UNSTATIONARY ARC COLUMN

Let us investigate into the dynamics of a DC arc, considering at the same time the transient conditions of electric supply systems with automatic control. The arc dynamics is determined by entalphy characteristics and the plasma flux moving in the cylindrical part of the column. Plasma flows from the conical part situated near the cathode to the cylindrical part. In the conical part, a portion of gas mass flux is dispersed beyond the boundary of the column. In modelling the dynamic parameters of arc the flux enthalpy can be neglected being a small quantity as compared to the enthalpy within the arc column area. The differential equation of volume density of energy flux in the cylindrical part of the unstationary arc column is:

$$E_c(\tau)^2 \cdot \sigma(T_c(r,\tau)) - \sigma_\varepsilon(T_c(r,\tau)) + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r \cdot \lambda(T_c(r,\tau))\frac{\mathrm{d}T_c(r,\tau)}{\mathrm{d}r}\right] = \frac{\partial h_v}{\partial \tau} \tag{1}$$

where: E_c, T_c are the electric field intensity and the mean plasma temperature in the cylindrical part of the column, respectively; σ is the conductivity; λ is the coefficient of heat conduction; σ_ε is the volume density of plasma radiation power, h_v is the volume density of plasma enthalpy.

The numerical solution of Equation (1) in the unstationary condition of linking arc [1] indicates that the rate of changes in specific enthalpy $\partial h_v/\partial\tau$ is proportional to the density of electric power converted into heat σE^2 in each point of the unstationary arc column (Fig. 1):

$$\frac{\partial h_v}{\partial \tau} = \frac{\partial\left\{\int_0^T \rho[T(r,\tau)] \cdot c_p[T(r,\tau)] \cdot \mathrm{d}T\right\}}{\partial \tau} = K_H \cdot \sigma[T(r,\tau)] \cdot E_c(\tau)^2 \tag{2}$$

where ρ is the density; c_p is the specific heat, K_H — is the proportional coefficient between the increase of enthalpy and plasma conductivity.

After linearization of the characteristics and assuming the proportionality:

$$\sigma_\varepsilon(T(r,\tau)) = K_\varepsilon \cdot \sigma(T(r,\tau)) \cdot E_c(\tau)^2 \tag{3}$$

Eqn. (1) assumes the following table form:

$$E_c(\tau)^2 \cdot \sigma(T_c(r,\tau)) \cdot (1 - K_\varepsilon - K_H) + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r \cdot \lambda(T_c(r,\tau))\frac{\mathrm{d}T_c(r,\tau)}{\mathrm{d}r}\right] = 0 \tag{4}$$

where K_ε — is the proportional coefficient, resulting from similarity of functions $\sigma_\varepsilon(T(r,\tau))$ and $\sigma(T(r,\tau))$.

It has an analytical solution analogical to the stationary arc solution [3]:

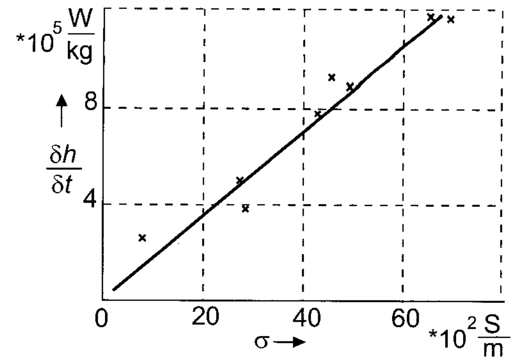


Fig. 1. Rate of Changes in Plasma Specific Enthalpy in the Column of an Unstationary Arc

$$S(\tilde{r},\tau) = S_p + \frac{p_\lambda(\tau) \cdot \left[S(\tilde{r},\tau) - S_p\right]}{2\pi z_1 \cdot J_1(z_1) \cdot \sigma[S(\tilde{r},\tau)]} \cdot \left[\left(\frac{\mathrm{d}\sigma}{\mathrm{d}S}\right)_{\tilde{r},\tau}\right] \cdot J_0(2{,}405\tilde{r},\tau) \tag{5}$$

where S_p is the value of heat flux potential at temperature T_p of the arc surface; J_0 is the Bessel function of zero order; J_1 is the Bessel function of the first order; $\tilde{r} = r/r_c$ is the relative radius; z_1 is the first pole of the Bessel function.

Taking into account the known function of plasma heat flux potential $S(T)$ the solution (5) can be represented as the sought distribution of plasma temperature in the cylindrical part of unstationary arc:

$$T_c(\tilde{r},\tau) = T[S(\tilde{r},\tau)] \tag{6}$$

At every moment of the transient process, solution (6) is uniquely associated with the value of linear power density $p_\lambda(\tau)$. This kind of power is connected with dissipating energy of electric field due to conducting heat by plasma in the cylindrical part of unstationary arc. As in the stationary condition [3], we shall now introduce some notions of universal physicothermal functions:

— plasma conductance function:

$$\tilde{g}(p_\lambda) = 2\pi\int_0^1 \sigma(T_c(\tilde{r})) \cdot \tilde{r}\mathrm{d}\tilde{r}, \quad \frac{\mathrm{S}}{\mathrm{m}} \tag{7}$$

— radiation function:

$$\tilde{P}_\varepsilon(p_\lambda) = 2\pi\int_0^1 \sigma_\varepsilon(T_c(\tilde{r})) \cdot \tilde{r}\mathrm{d}\tilde{r}, \quad \frac{\mathrm{W}}{\mathrm{m}^3} \tag{8}$$

— enthalpy function:

$$\tilde{H}(p_\lambda) = 2\pi\int_0^1 \rho(T_c(\tilde{r})) \cdot h(T_c(\tilde{r})) \cdot \tilde{r}\mathrm{d}\tilde{r}, \quad \frac{\mathrm{J}}{\mathrm{m}^3} \tag{9}$$

Figure 2 shows calculated universal enthalpy functions of argon, air, hydrogen, and nitrogen. At every moment of the transient process of the dynamic system the functions depend only on the value of linear power density $p\lambda(\tau)$.

An important conclusion follows from the solutions (5) and (6) the energy equation (1). If arcs are burning in gas of the same chemical composition and at a given moment of the transient process the values of linear power density are equal:

$$p_{\lambda}(\tau) = \frac{P_{\lambda}(\tau)}{l_a} = \text{const} \qquad (10)$$

then, the dependences between the instantaneous values of electrical, geometrical and thermal external characteristics of the column cylindrical part will be constant:

$$\frac{G_c \cdot l_a}{r_c^2} = \widetilde{g} = \text{const}$$

$$\frac{P_{\varepsilon}(\tau)}{r_c^2 \cdot l_a} = \widetilde{P}_{\varepsilon} = \text{const} \qquad (11)$$

$$\frac{H(\tau)}{r_c^2 \cdot l_a} = \widetilde{H} = \text{const}$$

where G_c is the instantaneous conductance of the cylindrical part of unstationary arc column brought to arc length l_a; H is the plasma enthalpy in the arc column.

Equations (11) are similarity conditions for the cylindrical part of the unstationary arc. They are valid for any arc or plasma device. If the values of linear power density $p\lambda(\tau)$ of arcs burning in gas of a given chemical composition are identical, then the similarity conditions (11), the solutions (5) and (6) and the universal functions (7) – (9) coincide for stationary and unstationary conditions. This property of the two-layer arc makes it possible to calculate non-linear functions being components of the known dynamic arc equation [1]:

$$\frac{dH(\tau)}{d\tau} = \frac{i^2}{G(\tau)} - P_{\Delta}(\tau) \qquad (12)$$

where i, $G(\tau)$ are the instantaneous values of current and conductance, respectively; $P_{\Delta}(\tau)$ is the instantaneous value of power losses during the heat exchange between the unstationary arc and surrounding environment.

3. THE INFLUENCE OF THE CONICAL PART OF THE COLUMN AND OF EXTERNAL DISTURBANCES ON THE DYNAMIC ARC MODEL

The power of electric field energy dissipated in the conical part of the column is used up on heating of plasma flux [4]:

$$P_{kon}(l_k) = IU_{kon} = I \cdot (E_k - E_c) \cdot r_c k_e \cdot \left[1 - \exp\left(-\frac{l_k}{r_c k_e}\right)\right] \qquad (13)$$

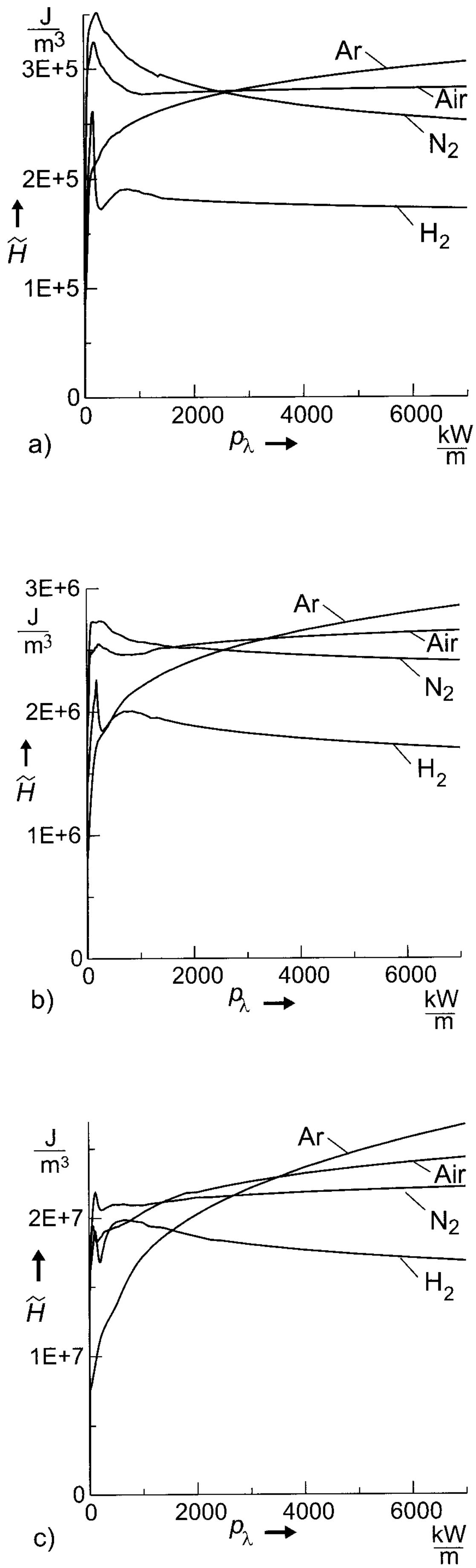


Fig. 2. Universal Functions of the Enthalpy of Argon, Air and Nitrogen: a) Pressure $p = 10^4$ Pa; b) Pressure $p = 10^5$ Pa; c) Pressure $p = 10^6$ Pa

where: I is the arc current; r_c is the radius of the arc cylindrical part; E_k is the electric field intensity near the cathode; k_e is the electric field intensity attenuation coefficient; l_k is the length of the conical part of the arc column ($l_k = (2 \div 3)\ r_c$); U_{kon} is the arc voltage, due to convective dissipation of heat.

Let the enthalpy of plasma in the column cylindrical part of length 1 m be marked as h_c. Then the plasma enthalpy in the conical part of the column will be:

$$H_k = h_c \cdot l_k - P_{kon} \cdot \frac{l_k}{\upsilon_k} \tag{14}$$

where: υ_k is the velocity of plasma at the exit from the conical part of the column.

The plasma enthalpy of the column cylindrical part of length l_a is:

$$H_c = h_c \cdot (l_a - l_k) + P_{kon} \cdot \frac{l_k}{\upsilon_k} \tag{15}$$

In this case the total plasma enthalpy of the arc:

$$H = H_k + H_c = h_c \cdot l_k - P_{kon} \cdot \frac{l_k}{\upsilon_k} + h_c \cdot (l_a - l_k) + P_{kon} \cdot \frac{l_k}{\upsilon_k} = h_c \cdot l_a \tag{16}$$

depends only on enthalpy function h_c in the cylindrical part and on arc length l_a.

According to the conditions of the two-layer heterogeneous arc model [3, 5] the values of instantaneous power satisfy the equation:

$$\frac{i^2}{G(\tau)} = i \cdot E_c(\tau) \cdot l(\tau) + i \cdot u_{kon}(\tau) + i \cdot u_{AK}(\tau) \tag{17}$$

where u_{kon} is the instantaneous voltage caused by convection in the conical part of the arc. In the modelling of dynamic systems with the arc, it is assumed that the sum of voltage drops near the electrodes $u_{AK}(\tau)$ is constant. It is identical as in the stationary condition U_{AK}:

$$u_{AK}(\tau) = U_{AK} \tag{18}$$

This simplifying assumption does not introduce any significant inaccuracy to the calculations of dynamic arc parameters. It is also assumed that the dependence of the instantaneous value of arc voltage convection component $u_{kon}(\tau)$ on current i is unique. The duration of relaxation of the convection process of electric field energy dissipation in the conical part of the column is a small quantity as compared to the unstationary unit of Eqn. (17). It depends on the rate of changes in the arc enthalpy (16):

$$\theta_{kon} = \frac{l_k}{\upsilon_k} << \theta_H \tag{19}$$

where θ_H is the relaxation time of radiation and conduction processes of heat dissipation of the arc column.

Because of that the dependence between the instantaneous values of convection voltage drop and current $u_{kon}(i)$ in the dynamic stage is the same as static function $U_{kon}(I)$ in the stationary condition of arc burning:

$$u_{kon}(i) \equiv U_{kon}(I) \tag{20}$$

Now, let us introduce a parameter of instantaneous conductance of the cylindrical part of unitary length:

$$g_c(\tau) = \frac{i}{E_c(\tau)} \tag{21}$$

After taking into account (18), (20) and (21) the equation of dynamic arc instantaneous power (17) becomes:

$$\frac{i^2}{G(\tau)} = \frac{i^2}{g_c(\tau)} l_a(\tau) + i \cdot u_{kon}(i) + i \cdot U_{AK} \tag{22}$$

The energy defined by Eqn. (22) covers heat fluxes $P_\Delta(\tau)$ and the change of enthalpy $H(\tau)$ of the dynamic system. Power of heat fluxes $P_\Delta(\tau)$ is determined by the processes of electric field energy dissipation in the dynamic arc occuring due to plasma heat conduction and radiation, as well as convection transport induced by plasma motion:

$$P_\Delta(\tau) = p_\lambda(\tau) \cdot l_a(\tau) + p_\varepsilon(\tau) \cdot l_a(\tau) + i \cdot u_{kon}(i) + i \cdot U_{AK} \tag{23}$$

In real dynamic arc systems constant gasodynamic disturbances $\Phi_p(\tau)$ occur. They cause hydrodynamic instability of the arc column surface. This surface is a boundary between plasma moving laminarly within the column and turbulent gas flux flowing around the arc column. Local vortices of turbulent gas cause reversible disturbances and lead to temporary changes in the arc surface [2]. As a result, there occur temporary changes in the balance of the arc energy. Power $P_\Delta(\tau)$ of heat fluxes shows periodic variations as compared to the stationary state. Another factor influencing the arc energy balance is disturbance $\Phi_l(\tau)$ caused by the change of arc length $l_a(\tau)$ during thermal, gasodynamic and electrodynamic processes. When (16) has been considered, the dynamic arc equation (12) can be transformed to the general form:

$$\frac{d[h_c(\tau) \cdot l_a(\tau)]}{d\tau} = \frac{i^2}{G_c(\tau)} - P_\Delta(\tau) \pm \Phi_p(\tau) \pm \Phi_l(\tau) \tag{24}$$

After differentiation (12) becomes:

$$l_a(\tau)\cdot\frac{\mathrm{d}h_c(\tau)}{\mathrm{d}\tau}+h_c(\tau)\cdot\frac{\mathrm{d}l_a(\tau)}{\mathrm{d}\tau}=$$

$$=\frac{i^2}{G_c(\tau)}-P_\Delta(\tau)\pm\Phi_p(\tau)\pm\Phi_l(\tau) \tag{25}$$

Substituting (22) and (23) into (25) we obtain the general equation of arc as a component of a dynamic system of a thermal device:

$$l_a(\tau)\cdot\frac{\mathrm{d}h_c(\tau)}{\mathrm{d}\tau}+h_c(\tau)\cdot\frac{\mathrm{d}l_a(\tau)}{\mathrm{d}\tau}= \tag{26}$$

$$=\frac{i^2}{g_c}\cdot l_a(\tau)-p_\lambda(\tau)\cdot l_a(\tau)-p_\varepsilon(\tau)\cdot l_a(\tau)\pm\Phi_p(\tau)\pm\Phi_l(\tau)$$

The practice of designing controlled feeding systems of electrothermal plasma devices [1, 4] shows that the methods of modelling arc dynamic systems taking into account local properties without disturbances $\Phi_p(\tau)$ and $\Phi_l(\tau)$ are sufficient for practical purposes. In the modelling of dynamic arc with the consideration of local properties the following conditions hold:

$$\Phi_p(\tau)=0; \qquad \Phi_l(\tau)=0; \qquad l_a(t)=l_a=\mathrm{const} \tag{27}$$

After taking (27) into account the dynamic arc equation (26) is:

$$\frac{\mathrm{d}h_c(\tau)}{\mathrm{d}\tau}=\frac{i^2}{g_c}-p_\lambda(\tau)-p_\varepsilon(\tau) \tag{28}$$

Non-linear thermal and electric functions $h_c(\tau)$, $p_\lambda(\tau)$, $p_\varepsilon(\tau)$ and g_c are components of Equation (28). If the assumption of plasma balance is considered, these functions depend uniquely on instantaneous temperature distribution $T(r,\tau)$ and instantaneous value of radius r_c of the cylindrical part of the dynamic arc column:

$$g_c(\tau)=2\pi\int_0^{r_c}\sigma(T_c(r,\tau))\cdot r\mathrm{d}r \tag{29}$$

$$h_c(\tau)=2\pi\cdot\int_0^{r_c}\rho[T_c(r,\tau)]\cdot h[T_c(r,\tau)]\cdot r\mathrm{d}r \tag{30}$$

$$p_\varepsilon(\tau)=2\pi\cdot\int_0^{r_c}\sigma_\varepsilon[T_c(r,\tau)]\cdot r\mathrm{d}r \tag{31}$$

$$p_\lambda(\tau)=2\pi\int_0^{r_c}-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\cdot\lambda(T_c(r,\tau))\frac{\mathrm{d}T_c(r,\tau)}{\mathrm{d}r}\right)\mathrm{d}r \tag{32}$$

where ρ is the density.

Consequently, non-linear functions $h_c(\tau)$, $p_\lambda(\tau)$ and $p_\varepsilon(\tau)$ in Eqn. (28) can be represented as unique functions of instantaneous conductance g_c (21):

$$\frac{\mathrm{d}h_c(g_c)}{\mathrm{d}\tau}=\frac{i^2}{g_c}-p_\lambda(g_c)-p_\varepsilon(g_c) \tag{33}$$

After differentiating (33) we obtain the final form of the dynamic arc equation:

$$\frac{\mathrm{d}h_c(g_c)}{\mathrm{d}g_c}\frac{\mathrm{d}g_c}{\mathrm{d}\tau}=\frac{i^2}{g_c}-p_\lambda(g_c)-p_\varepsilon(g_c) \tag{34}$$

Non-linear functions $h_c(g_c)$, $p_\lambda(g_c)$ and $p_\varepsilon(g_c)$ in Eqn. (34) can be calculated from the equations of stationary arc burning in the designed device:

$$h_c(g_c)\equiv[h_c(g_c)]_{stat}$$

$$p_\lambda(g_c)\equiv[p_\lambda(g_c)]_{stat} \tag{35}$$

$$p_\varepsilon(g_c)\equiv[p_\varepsilon(g_c)]_{stat}$$

The integration of the dynamic arc equation (34) gives instantaneous conductance g_c (21) which is related to instantaneous conductance G_a of the dynamic arc by the following:

$$G_a=\frac{i}{u_a}=\frac{1}{\frac{l_a}{g_c}+\frac{U_{AK}+u_{kon}(i)}{i}} \tag{36}$$

4. THE TRANSMITATIONS OF ARC AS A COMPONENT OF A CONTROLLED FEEDING SYSTEM OF ARC AND PLASMA DC DEVICES

The dynamic arc is treated as a component of a controlled current supply system of an electrotechnological device. In this case it is a dipole in which the instantaneous value of voltage at the input is u_a, and the instantaneous current at the output is i. Transmittance $W_{u,i}$ (s) of the dipole is a local dynamic voltage-current characteristic of the arc. In the analysis we will take into consideration the principle of superposition of the separate components of electric field dissipated energy [3]. Then, the instantaneous value of arc voltage contains three components:

$$u_a = u_c + u_{kon} + U_{AK} \quad (37)$$

where u_c is the instantaneous value of voltage drop conditioned by the dissipation of electric field energy occurring due to heat conduction and radiation of column plasma:

$$u_c = l_a \cdot E_c(\tau) \cdot i = \frac{i}{g_c} \cdot l_a \quad (38)$$

If the disturbance is small, the components of the voltage drop are:

$$u_a = U_a + \Delta u_a\,;\ u_c = U_c + \Delta u_c\,;\ u_{kon} = U_{kon} + \Delta u_{kon} \quad (39)$$

where U_a, U_c, U_{kon} are the components of the arc voltage in the equilibrium point of the dynamic system with current $I = i_0$.

Let us introduce the symbols of relative deflections of the voltage components:

$$\Delta\overline{u}_a = \frac{\Delta u_a}{U_c};\ \Delta\overline{u}_c = \frac{\Delta u_c}{U_c};\ \Delta\overline{u}_{kon} = \frac{\Delta u_{kon}}{U_c} \quad (40)$$

Taking into account the local linearization of function $u_{kon}(i)$ (20) we obtain:

$$\Delta u_{kon} = \left[\frac{\mathrm{d}u_{kon}(i)}{\mathrm{d}i}\right]_{i_0} \cdot \Delta i = {}_d R_{kon} \cdot \Delta i \quad (41)$$

where ${}_dR_{kon}$ is the dynamic resistance caused by convection heat transfer. After dividing (41) by voltage drop $U_c = \frac{i_0}{g_c(i_0)} \cdot l_a$ we arrive at:

$$\Delta\overline{u}_{kon} = \left[\frac{\mathrm{d}u_{kon}(i)}{\mathrm{d}i}\right]_{i_0} \cdot \frac{g_c(i_0)}{l_a} \cdot \Delta\overline{i} = k_k \cdot \Delta\overline{i} \quad (42)$$

where $\Delta\overline{i} = \frac{\Delta i}{i_0}$ is the relative local deflection of arc current.

Considering formulae (37), (39) and (40) we can represent the relation between the relative deflections of arc voltage components as:

$$\Delta\overline{u}_a = \Delta\overline{u}_c + \Delta\overline{u}_{kon} = \Delta\overline{u}_c + k_k \cdot \Delta\overline{i} \quad (43)$$

Now, we shall introduce the notion of transmittance $W_s(s)$ pertaining to function $\Delta\overline{u}_0$ of arc voltage:

$$W_s(s) = \frac{\Delta\overline{u}_c}{\Delta\overline{i}} = \frac{\Delta\overline{E}_c}{\Delta\overline{i}} \quad (44)$$

The condition (19) on the static properties of function $u_{kon}(i)$ is assumed in our reasoning. According to (43) the local dynamic voltage-current characteristic of arc has the form of transmittance $W_{u,i}(s)$:

$$W_{u,i}(s) = W_s(s) + k_k \quad (45)$$

Transmittance $W_s(s)$ is obtained by means of a local linearization method of the dynamic arc equation (34). Now we shall introduce the concept of relative local deflection of instantaneous conductance $g_c = g_c(i_0) + \Delta g_c$:

$$\Delta\overline{g}_c = \frac{\Delta g_c}{g_c(i_0)} \quad (46)$$

where $g_c(i_0)$ is the static conductance in equilibrium point of the dynamic system with arc current i_0. After summing the function on the right-hand side of Eqn. (34):

$$p_\lambda(g_c) + p_\varepsilon(g_c) = p_c(g_c) \quad (47)$$

we arrive at:

$$\frac{\mathrm{d}h_c(g_c)}{\mathrm{d}g_c}\frac{\mathrm{d}g_c}{\mathrm{d}\tau} = \frac{i^2}{g_c} - p_c(g_c) \quad (48)$$

Let us perform local linearization of functions $h_c(g_c)$ and $p_c(g_c)$:

$$h_c(g_c) = h_c(i_0) + \left[\frac{\mathrm{d}h_c(g_c)}{\mathrm{d}g_c}\right]_{i_0} \cdot [g_c - g_c(i_0)] = = h_c(i_0) + k_h \cdot [g_c - g_c(i_0)] \quad (49)$$

$$p_c(g_c) = p_c(i_0) + \left[\frac{\mathrm{d}p_c(g_c)}{\mathrm{d}g_c}\right]_{i_0} \cdot [g_c - g_c(i_0)] = = p_c(i_0) + k_p \cdot [g_c - g_c(i_0)] \quad (50)$$

Subsequently, a number of simple mathematical operations have been performed. Equations (49) and (50) were substituted into (48), which was then differentiated. In the resulting equation the small quantities of secondary order were neglected and the stationary arc equation was substracted from it. In this way we obtain:

$$g_c(i_0) \cdot k_h \cdot \frac{\mathrm{d}\Delta g_c}{\mathrm{d}\tau} + p_c(i_0) \cdot \Delta g_c + k_p \cdot g_c(i_0) \cdot \Delta g_c = 2 i_0 \cdot \Delta i \quad (51)$$

Let us divide Eqn. (51) by static conductance $g_c(i_0)$:

$$g_c(i_0) \cdot k_h \cdot \frac{\mathrm{d}\Delta\overline{g}_c}{\mathrm{d}\tau} + \left[p_c(i_0) + k_p \cdot g_c(i_0)\right] \cdot \Delta\overline{g}_c = \frac{2 i_0^2}{g_c(i_0)} \cdot \Delta\overline{i} \quad (52)$$

Let us transform Eqn. (52) to the standard form:

$$\frac{g_c(i_0)\cdot k_h}{p_c(i_0)+k_p\cdot g_c(i_0)}\cdot\frac{\mathrm{d}\Delta\bar{g}_c}{\mathrm{d}\tau}+\Delta\bar{g}_c=$$

$$=\frac{2i_0^2}{\left[p_c(i_0)+k_p\cdot g_c(i_0)\right]\cdot g_c(i_0)}\cdot\Delta\bar{i} \qquad (53)$$

We shall now introduce symbols of coefficients in the linearized equation (53) of the dynamic arc:
— time constant of the dynamic arc

$$\Theta_c=\frac{g_c(i_0)\cdot k_h}{p_c(i_0)+k_p\cdot g_c(i_0)} \qquad (54)$$

— transmittance factor of the dynamic arc

$$k_c=\frac{2i_0^2}{\left[p_c(i_0)+k_p\cdot g_c(i_0)\right]\cdot g_c(i_0)} \qquad (55)$$

According to (53) we obtain the dynamic arc transmittance:

$$W_c(s)=\frac{\Delta\bar{g}_c}{\Delta\bar{i}}=\frac{k_c}{\Theta_c\cdot s+1} \qquad (56)$$

The relation between the relative increments of the dynamic arc parameters can be represented as:

$$\Delta\bar{g}_c=\Delta\bar{i}-\Delta\bar{u}_c \qquad (57)$$

After substituting (57) into Eqn. (53) we can write:

$$\Theta_c\cdot\frac{\mathrm{d}\Delta\bar{u}_c}{\mathrm{d}\tau}+\Delta\bar{u}_c=\Theta_c\cdot\frac{\mathrm{d}\Delta\bar{i}}{\mathrm{d}\tau}+(1-k_c)\cdot\Delta\bar{i} \qquad (58)$$

According to (58), transmittance $W_s(s)$ (44) of the dynamic arc has the following form:

$$W_s(s)=\frac{\Theta_c\cdot s+(1-k_c)}{\Theta_c\cdot s+1} \qquad (59)$$

In this way, on the basis of (45) and (59), the resultant dynamic arc transmittance $W_{u,i}(s)$ in the form of external local voltage-current characteristic can be represented as:

$$W_{u,i}(s)=\frac{\Theta_c\cdot s+(1-k_c)}{\Theta_c\cdot s+1}+k_k=$$

$$=\frac{\Theta_c\cdot(1+k_k)\cdot s+(1-k_c+k_k)}{\Theta_c\cdot s+1} \qquad (60)$$

As it follows from (60) the dynamic arc as a component of a controlled system with electric supply can be represented by two typical parallel branches:
— a real differentiating branch

$$W_1(s)=\frac{\Theta_c\cdot(1+k_k)\cdot s}{\Theta_c\cdot s+1} \qquad (61)$$

— and an inertial branch of the first order

$$W_2(s)=\frac{1-k_c+k_K}{\Theta_c\cdot s+1} \qquad (62)$$

We shall represent arc transmittance factors (60) in the elaborate form and introduce new symbols of coefficients:

$$\Theta_a=\Theta_c\cdot(1+k_k)= \qquad (63)$$

$$=\frac{g_c(i_0)\cdot\left[\frac{\mathrm{d}h_c(g_c)}{\mathrm{d}g_c}\right]_{i_0}}{p_c(i_0)+\left[\frac{\mathrm{d}p_c(g_c)}{\mathrm{d}g_c}\right]_{i_0}\cdot g_c(i_0)}\cdot\left\{1+\left[\frac{\mathrm{d}u_{kon}(i)}{\mathrm{d}i}\right]_{i_0}\cdot\frac{g_c(i_0)}{l_a}\right\}$$

$$k_a=k_c-k_k= \qquad (64)$$

$$=\frac{2i_0^2}{p_c(i_0)\cdot g_c(i_0)+\left[\frac{\mathrm{d}p_c(g_c)}{\mathrm{d}g_c}\right]_{i_0}\cdot g_c^2(i_0)}-\left[\frac{\mathrm{d}u_{kon}(i)}{\mathrm{d}i}\right]_{i_0}\cdot\frac{g_c(i_0)}{l_a}$$

The notions of time-constant $\Theta_a=\Theta_c\cdot(1+k_k)$ and of factor $k_a=k_c-k_k$ are introduced. In this case transmittance $W_{u,i}(s)$ assumes the simpler, known form [1, 5]:

$$W_{u,i}(s)=\frac{\Theta_a\cdot s+(1-k_a)}{\Theta_c\cdot s+1} \qquad (65)$$

In accordance with transmittance form (65) the differential equation of the dynamic arc as a component of a dynamic system is similar to (58):

$$\Theta_c\cdot\frac{\mathrm{d}\Delta\bar{u}_a}{\mathrm{d}\tau}+\Delta\bar{u}_a=\Theta_a\cdot\frac{\mathrm{d}\Delta\bar{i}}{\mathrm{d}\tau}+(1-k_a)\cdot\Delta\bar{i} \qquad (66)$$

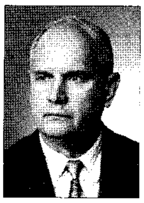
In the above equation variables i_0, $g_c(i_0)$ denote the steady values of current and the conductance of the arc cylindrical part at the equilibrium point of the dynamic system. According to conditions (19) and (35), nonlinear functions $h_c(g_c)$, $p_c(g_c)=p_\lambda(g_c)+p_\varepsilon(g_c)$, $u_{kon}(i)$ and their derivatives are components of Eqn. (66). They can be calculated in the stationary operating conditions of the designed arc or plasma device.

5. CONCLUSIONS

The present paper has demonstrated that the solutions of energy equations and arc similarity conditions in stationary and unstationary states are, in fact, convergent. Because of that the resultant transmittance of the locally linearized arc, depending on power characteristics of gases, could be obtained. The dynamic arc equation (66), unlike other existing mathematical models, takes into account the parameters of all the heat exchange processes between the arc column and the surrounding environment, such as radiation, plasma heat conductivity, and convection resulting from plasma motion. The developed mathematical model of the dynamic arc will provide the basis for specifying the classical Kaufmann condition—a typical problem of non-linear electrotechnology and automatics.

REFERENCES

1. Кручинин А.М.: *Расчет динамических систем с электрической дугой. Издательство МЭИ.* Москва 1988. (Calculating dynamic systems with an electric arc.—in Russian).
2. Кручинин А.М.: *Некоторые результаты исследования автоматической системы стабилизации тока электродуговых нагревателей газа* (Selected results of research on the automatic system of stabilisation of current in arc gas heaters—in Russian). Изв. СО АН СССР, сер. техн. наук, 1967, вып 1, № 3, с. 15-23.
3. Kruczinin A.M., Sawicki A.: *Urządzenia elektrotechnologiczne z nagrzewaniem łukowym i plazmowym, cz.1. Teoria nagrzewania łukowego i plazmowego.* Skrypt 17, Wydawnictwo Politechniki Częstochowskiej, Częstochowa 1997. (Electrotechnological devices with arc and plasma heating, part 1. Theory of arc and plasma heating—in Polish).
4. Kruczinin A.M., Sawicki A.: *Piece i urządzenia łukowe. Seria Monografie nr 74, Wydawnictwo Politechniki Częstochowskiej, Częstochowa 2000.* (Arc furnaces and devices—in Polish)
5. *Автоматическое управление электротермическими установками.* Под ред. А.Д. Свенчанского. Изд-во Энергоатомиздат, Москва 1990. (Automatic control of electrothermal devices—in Russian)



Anatoli M. Krouchinin

Anatoli M. Krouchinin was born on January 7, 1938, in Vyazma (Smolen County), Russia. He graduated from Moscow Power Engineering Institute (MPEI), specializing in electrothermal devices, in 1961 and received the PhD degree in 1967. In 1982 he defended a dissertation qualifying him for degree of Doctor of Science in Engineering.

Prof. Krouchinin is a corresponding member of Russian Academy of Electrical Science. He is the Chairman of Electric Heating Engineering Section of the Russian Society of Science, a member of Electric Heating Engineering Board of Russia. He is an author or co-author of more than 130 publications in periodicals and conference proceedings, published in Polish, Russian and other languages, 18 monographs and textbooks on electric heating engineering, and 57 patents. He is currently working at the Institute of Electrotechnology, Technical University of Częstochowa, Poland.

Antoni Sawicki

Antoni Sawicki was born on July 22, 1952, in Częstochowa, Poland. He graduated from the Department of Electrical Engineering, Częstochowa Technical University, in 1977 and the Ph.D. degree from Moscow Power Engineering Institute (MPEI), Moscow, Russia in 1988. In 2000 he defended a dissertation qualifying him for degree of Doctor of Science in Engineering.

Prof. Sawicki is a member of the Association of Polish Electricians. He has been a member of the Board of Katowice Division of Polish Electrothermics Committee since 1997 and of the Board of Polish Electrothermics Committee since 1998. He is an author or co-author of more than 120 publications in periodicals and conference proceedings, published in Poland and abroad. He is currently working at the Institute of Electrotechnology, Technical University of Częstochowa, Poland.