

A DIGITAL SYSTEM TO MEASURE FREQUENCY AND AMPLITUDE OF POWER GRID VOLTAGE

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Summary: The paper presents complete, software-hardware solution of real-time measurement of frequency and amplitude of energetic voltage signal based on DSP processor. The hardware contains DSP board with A/D converter and 8051-based daughterboard with LCD and small keyboard. The software consists algorithm approximating sampled data of the voltage signed by a sinus function. This algorithm uses modified steepest descent method for error function minimization.

An innovative modification was developed in order to reduce direction oscillations in the steepest descent method. The modification increases the speed of function minimization by factor between 4 and 22. Results such as speed and accuracy are far better than in counting method of frequency measurement. Speed of about one measurement per second was achieved using a 40 MHz TMS320C32 DSP processor. The maximum relative error of frequency measurement amounted to 0.56% at 60 samples of measured signal and 0.18% at 100 data samples.

Key words: frequency measurement, amplitude measurement, DSP, signal processing, realtime system, function minimization, gradient method, steepest descent method, approximation

1. INTRODUCTION

A frequency and amplitude estimation algorithm for the power grid basic harmonic was proposed in the paper. The algorithm was then implemented and its performance verified in a measuring system employing a Texas Instruments TMS320C32 signal processor.

To determine the frequency and amplitude a method of approximating the signal by sinus function was used. The method may be used to measure the frequency of distorted signals and, thanks to this, can be applied in industrial measurements.

In the paper there is also described an innovatory solution that reduces directional oscillations produced during minimization by the steepest descent method. The solution accelerated significantly the performance of the algorithm (the number of iterations was decreased 4 to 22 times.) [5].

The aim of the action undertaken was to build a real-time working device. The system performance speed of about 1 measurement per second was reached. The accuracy was approved as satisfactory. The frequency measurement uncertainty of about 0.23% was obtained for measurement of limiting distortions (THD 8%) permitted by the Polish Standard PN-EN50160.

2. SYSTEM DESCRIPTION

The presented measuring system consists of the hardware and software parts.

The basic element of the hardware part is a board with a 40 MHz Texas Instruments TMS320C32 signal processor in which the algorithm of parameter estimation of the signal is implemented. Samples of the signal are taken by a 14-bit compensation A/C converter, and then, through a serial port, they are transmitted into the memory of the processor performing calculations. The input voltage variability of the A/C processing path amplifier covers the range from -1.8V to +1.8V.

Address decoder made on a Lattice ispLSI1016E programmable logic circuit is responsible for communication with external memory and other external devices. The board contains also an external memory of the capacity 64,000 32-bit words, EPROM memory of 8-bit organization containing the board software and a 50-pin communication connection with external devices through which the results of parameter estimation of the signal are transmitted to the second board.

Performance of the second board is based on an Atmel single-system microcontroller (clone 8051). The task of the board is to operate a 16-key keyboard and an LCD display where measurement results are displayed.

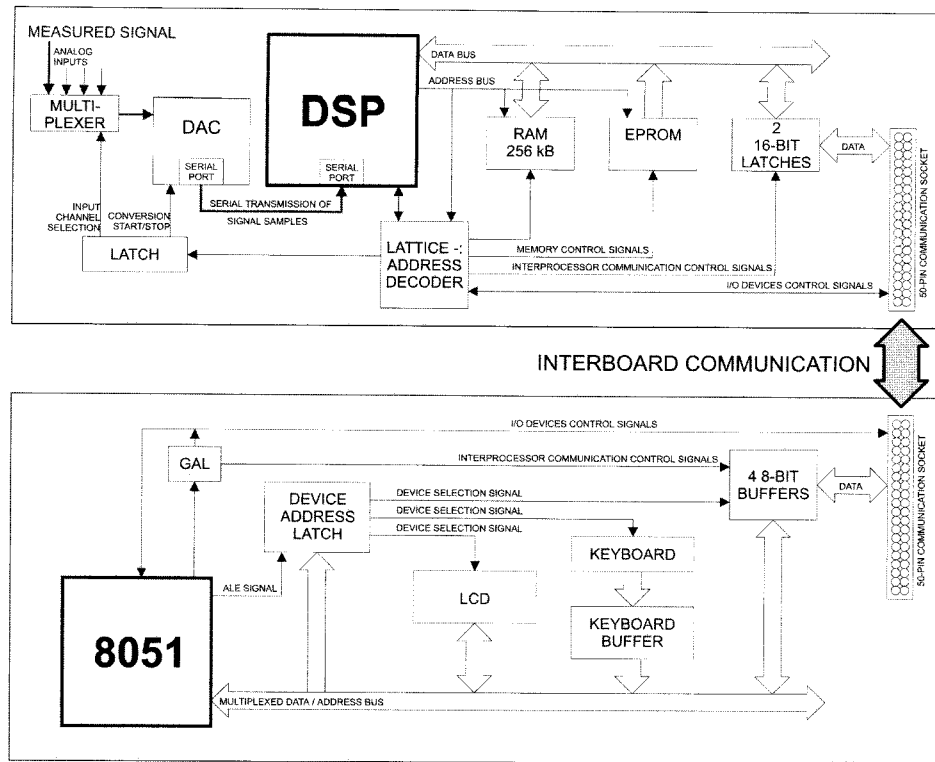


Fig. 1. Schematic Diagram of Measuring System

The interboard communication is of master-slave type with the signal processor board always as the master. Interruptions of both processors are used for communication. More than one external device can be connected to the 50-pin connection on the DSP board.

A schematic of the measuring system is presented in Figure 2.

3. MEASURING METHOD

To determine the power grid voltage frequency and amplitude a method of approximating the samples of the signal under consideration by a sinus signal of known parameters was used.

This method was selected in order to make it possible to measure parameters of distorted signals and to avoid some inconveniences concerned with signal frequency measurements by the pulse counting method used in most devices¹.

Compared with pulse counting method, a single measurement in the presented system lasts shorter at a given accuracy, which enables also nonstationary signals to be measured. The system makes it possible estimation of the amplitude of the basic harmonic and work with distorted signals.

3.1. Approximation algorithm

After each run of the main loop of the program a predetermined number of samples of the measured signal that are to

be approximated by a sinusoid with given parameters is drawn at 1 kHz frequency (which enables analysis of a signal with spectrum limited to the 10th harmonics) and stored in the DSP memory.

To a single approximation 60 samples are used represented by a vector y_1, y_2, \dots, y_N of length $N = 60$ values distant one from another by a constant period $\Delta t = 0.001$ s. The samples can be approximated by the sinusoid given by equation (1).

$$y(t) = A_0 \sin(\omega_0 t + \varphi_0) \quad (1)$$

A_0 denotes the amplitude of the approximating function, ω_0 is angular frequency of the approximating function, φ_0 is its phase², while t is the time counted from the instant of drawing the first sample (the beginning of the current measurement). Discrete time instants t_1, t_2, \dots, t_N correspond to samples y_1, y_2, \dots, y_N and we assume that $t_0 = 0$ at the moment the measurement starts, i.e., when $t = 0$.

The approximation consists in determining such values of parameters A_0 , ω_0 and φ_0 of the sinusoid that it would represent best the measured signal. When the approximating function is, according to the adopted criterion, sufficiently close to the approximated samples of the measured signal, then it can be accepted that the current parameters of the approximating function (1) are close to real parameters of the signal and calculations can be finished.

1) These devices exhibit linear relationship between the accuracy and measurement time, and they measure the average signal frequency only during the pulse counting period, which is a significant obstacle in the case of nonstationary signals.

2) In this paper, the signal phase is understood as the distance measured in time or angular units between the instant when sampling of the signal under study begins and its first zero-crossing, i.e., its passing from negative to positive values.

To assess the quality of the approximation the mean square criterion [2] was used. That approximation is best whose sum of squares of distances between the values of the samples of the measured signal and the corresponding approximating function values is least. This sum, called further error function, is of the following shape

$$S(A, \omega, \varphi) = \sum_{i=1}^N [A \sin(\omega t_i + \varphi) - y_i]^2 \quad (2)$$

where known t_i 's and y_i 's are the corresponding pairs of time instants when the signal was sampled and the signal values at these instants.

Estimation of parameters of the measured signal means minimization of the error function (2).

3.2. Error function minimization

Minimization of the error function (2) is carried out by the steepest descent method. Thanks to this, analytically known forms of the derivatives of the minimized functions were used. The directional minimization was made by golden section of the segment [1, 3].

Performance of the steepest descent method consists in determining searching direction opposite to the gradient of the function minimized at a given point. Then a point is found in the direction minimization loop at which the value of the error function is the smallest. At this point next direction is determined and this cycle is repeated until the loop ending conditions are met [6].

4. IMPLEMENTATION OF THE ALGORITHM

Signal parameter estimation in this paper was divided into two stages: determination of frequency and phase³ of the measured signal by minimization of error function (3) of two variables, and then determination of signal amplitude. This was possible because the amplitude A_0 of the measured signal was independent of the two other parameters.

Initially assumed values of the coefficients of the accepted approximating function (1) range from 45 Hz to 55 Hz for frequency, from 0 msec to 22.4 msec for phase and from 0.8 V to 1.8 V for amplitude (at the input of the A/C processing path).

4.1. Initial calculations

The algorithm starts from collecting samples of the measured signal. Then the samples are normalized, and thanks to that the amplitude of the measured signal can be unit and the error function "becomes" the two-variable function:

$$S(\omega, \tau) = \sum_{i=1}^N [\sin(\omega(t_i + \tau)) - y_i]^2 \quad (3)$$

where t is a time equivalent of phase 9.

One or two minima may occur in the assumed variability domain of model coefficients (this is connected with the periodicity of the sinusoidal signal), and because of that such new coefficient variability domains are determined which are to include the global minimum only⁴.

4.2. Main loop of the program

In the main loop of the program the values of ω_0 and τ_0 are determined that are estimates of the real parameters of the measured signal. Minimization of the error function (3) starts in the middle of the rectangular searching area. Based on the analytical forms of the derivatives of the error function (3) the travel direction opposite to the gradient \vec{g} of the minimized function is determined.

After the travel direction is determined, the direction minimization loop is performed by the method of golden section of the segment. It takes place along the segment connecting the point at which the direction was determined with the point of intersection of the travel direction and the border of the searching area (the function (3) unimodal is in this area). The exit from the direction minimization loop is effected when the distance between the both segment ends is less than a predetermined value.

Then the next travel direction is determined and next run of the loop is run. The execution of the main minimization loop is stopped at a point where the gradient of the function (3) is close to zero to a given accuracy. This point is the searched minimum.

4.3. Determining amplitude

After minimum of the function (3) is found, i.e., after parameters ω_0 and τ_0 are determined, the error function becomes a function of single variable (i.e., amplitude) and assumes the shape:

$$S(A) = \sum_{i=1}^N [A \sin(\omega_0(t_i + \tau_0)) - y_i]^2 \quad (4)$$

The sampled values of the initial signal before normalization are restituted and the minimum of the error function (4) corresponding to the amplitude A_0 of the measured signal is determined by the classic least square method.

4.4. Problem of directional oscillation

Simulation study on the algorithm showed that the applied method of the steepest descent converges slowly. To attain the minimum, the algorithm makes many steps not directed at the minimum as is shown in Fig. 2.

It turned out during tests that after performing certain number of direction minimization loops the error function values at the points in between the beginning and the end of

1) Although the phase φ_0 of the signal is not measured variable it must be determined because it is an unknown coefficient of relationship (2).
2) Minimization with constraints was applied because the error function contains many minima and the global minimum searching methods [6] are not fast enough to this end.

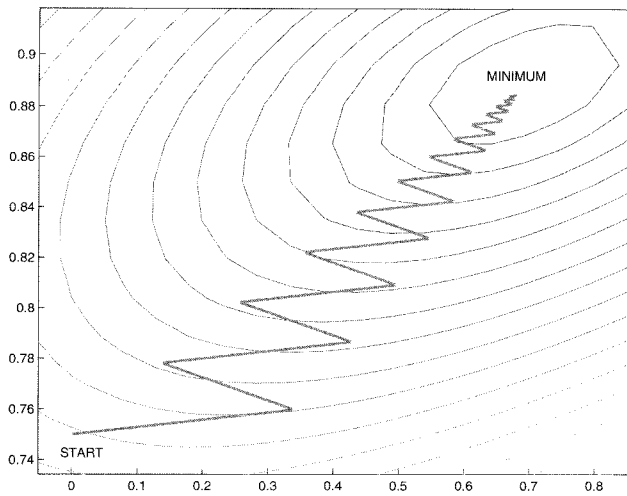


Fig. 2. An Example of Oscillation: Attaining Minimum Requires 90 Iterations

the contracted segment are equal (while the derivatives at these point change). This is caused by the limiting accuracy of machine calculations [5].

Identical values of the error function mean that the function is flat along the given segment and the golden section method is not able to determine the minimum location more accurate. After this moment is attained it is known only that the minimum is somewhere in between the ends of the contracted segment. The location of the minimum is assumed as the point in the middle of the ends of the segment.

The inaccurate determination of the location of the minimum by the direction minimization method results in the fact that the gradient determined at this point is often almost perpendicular to the direction of the valley that leads to the global minimum. Even very small distances of the gradient determination point from so called valley bottom produce significant oscillations of the minimum searching direction.

4.5. Method of reducing oscillation

To improve the convergence of the algorithm and therefore to reduce the time of the individual measurement, a modification to the steepest descent method was developed. The modification consists in verification of the angle between the subsequent travel directions. If the angle less than a certain set value repeats successively a few times then this is recognized as an occurrence of oscillations. Then the travel direction is corrected as to be close to the direction the valley descends.

Let us assume that the successive point pos_i at which the gradient \vec{g}_i is determined in i -th minimization step is connected with the preceding point pos_{i-1} by vector v_i with sense $-\vec{g}_i$, as is shown in Figure 3. Based on the knowledge on the vector v_{i-1} from i -th step and the position pos_{i-1} from $(i-1)$ -th step the angle α_i is calculated between vectors v_{i-1} and v_i . To calculate the angle the following equation is used:

$$\alpha_i = \arctan \left(\frac{v_{i-1}^x v_i^y - v_i^x v_{i-1}^y}{v_{i-1}^x v_i^x + v_{i-1}^y v_i^y} \right) \quad (5)$$

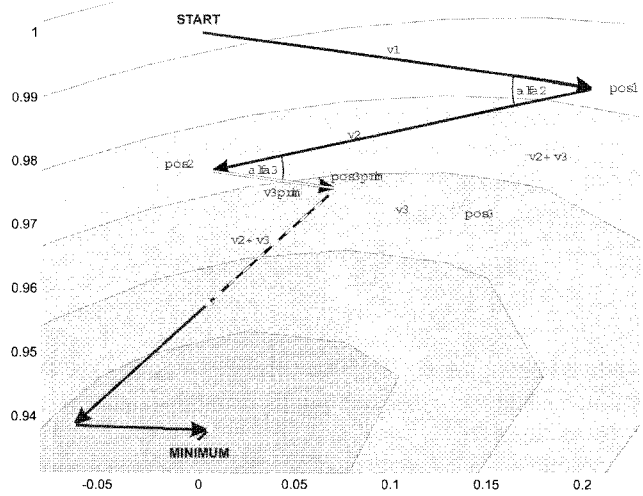


Fig. 3. Calculation of the Angle Between the Vectors, pos_3 and v_3 are Corrected Position pos_3 and Vector v_3

where v_i^x and v_i^y denote the first and second coordinates of i -th vector, respectively.

When the angle α_1 is less than 1.55 rad, i.e. 88.8 degrees the event counter is increased otherwise the counter is set to zero.

After three such events occur in three subsequent steps the oscillation occurrence is assumed and a correction of the direction takes place. In next step gradient is not calculated but the direction v_{i+1} is defined as vector sum $v_{i-1} + v_i$. This vector pointing the travel direction is fixed in the middle of the length of vector v_i .

The admissible angle between direction is changed after first occurrence of oscillation to 0.6 radian i.e., 34.4 degrees. Also, the number of events necessary for directional correction changes. These values were selected experimentally.

Thanks to directional correction a significant increase in speed of algorithm performance was achieved. The number of iteration needed for finding the minimum was reduced by 4 to 22 times. A result of directional correction for a typical measurement is shown in Figure 4.

5. ASSESSMENT OF SYSTEM PERFORMANCE

In order to assess the measurement accuracy such input signals were chosen only which could be available both in the Matlab environment and at the output of the laboratory function generator HP3325B whose accuracy of generated signal frequency was 0.005% of the predetermined value.

5.1. Test types

Test signals were sampled with 1 kHz frequency. Sampling could be started at an arbitrary point of the measured signal. The frequency of 50 Hz was assumed as the rated frequency of the measured signal. The test were carried out for a series of 60 or 100 samples of the measured signal, which corresponds to three or five periods of a 50 Hz signal.

The following test were made for various forms of the measured signal:

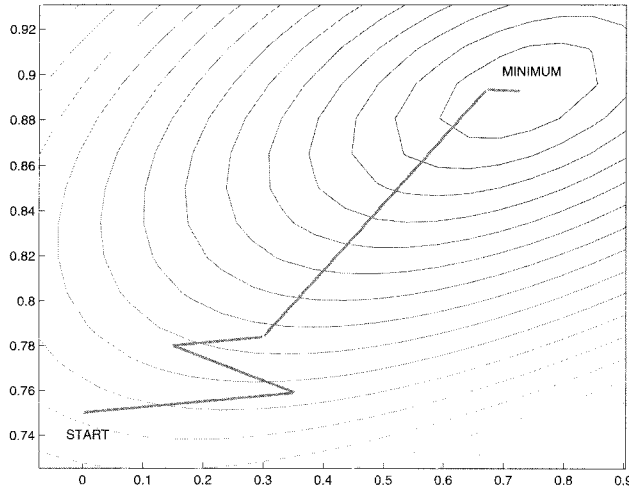


Fig. 4. Reducing Oscillation: Attaining Minimum Requires 7 Iterations.

1. No-distorted sinusoid test. The test signal was a undistorted sinusoid of frequency ranging from 45 to 55 Hz.
2. Third-harmonic test. The test signal was a 50 Hz sinusoid to which third harmonic was added of amplitude equal to 5% of the basic harmonic amplitude and phase lag ranging from 0 to 360 degrees with a step each 15 degrees.
3. Gaussian noise test. The test signal was a sinusoid to such noise was added that the test signal THD be between 7.8% and 8%.
4. Uniform-distribution noise test. The test signal was a sinusoid to such noise was added that the test signal THD be between 7.6% and 8%.
5. Amplitude modulation test. The test signal was a 150 Hz sinusoid modulated by a 50 Hz sinusoid for phase lag ranging from 0 to 360 degrees with a step each 15 degrees. Measurements were made for modulation depth 20%, 10%, 5% and 3%.
6. Triangle test was designed to determine the accuracy of frequency measurement of a signal containing many harmonics.
7. Rectangle test was designed also to determine the accuracy of frequency measurement of a signal containing many harmonics. The last two test were carried out for frequencies 48, 50 and 52 Hz.

In the study on a real system tests differing a bit from those used in simulation were applied because the available equipment did not allow to generate some of test signals mentioned above. In both types of study tests No. 1 and 6 were employed. In simulation study tests No. 1, 2, 3, 4, 6 were used while in device testing tests No. 1, 5, 6, 7.

Tests made in simulation study was aimed at accurate determination of measurement uncertainty of signals meeting the conditions specified in Polish Standard PN-EN50160.

5.2. Measurement errors

For signals whose distortions do not exceed the levels defined by Polish Standard PN-EN50160, the uncertainty of frequency measurement in simulation study is 0.53% for 60 samples of the measured signal and 0.23% for 100 samples of the measured signal.

Test No. 5 signal (amplitude modulation of 5% depth) is close to test No. 2 signal (sinusoid with 3rd harmonic added). The uncertainty of frequency measurement in the real system is for this signal 0.26% for 60 samples of the measured signal.

The results of simulation study prove that extending measuring time by 60% reduces the measurement uncertainty by half.

The measurement uncertainty of the basic harmonics is greater than that for frequency measurement. For signals whose distortions do not exceed the levels defined by Polish Standard PN-EN50160, the uncertainty of amplitude measurement in simulation study is 3% for 60 samples of the measured signal and 2.45% for 100 samples of the measured signal.

5.3. Measurement duration

The time of calculations connected with implementation of the presented algorithm in Matlab using a PC with 375 MHz Celeron processor is about 1.5 seconds, graph option disabled.

For 40 MHz TMS320C32 DSP, an average calculation time is ca. 0.7 sec for 60 samples of the measured signal. However, taking into account a low rate of frequency changes and small range of variability of power grid frequency, this speed is sufficient. When the number of analyzed samples increased up to 100 samples of the measured signal, the calculation time in DSP increased to about 1.2 sec but the measurement accuracy also increased.

5.4. Operation of the device

After the device is started, the measurement algorithm is copied from the EPROM memory to the DSP memory, and then it gets starting. When initial calculation are made, measuring frequency and amplitude of the signal begins. The algorithm operates in infinite loop and writes out the results on the LCD display. These include: frequency in Herzs, amplitude in Volts and time in milliseconds between the beginning of signal sampling and its first zero-crossing from negative to positive values.

To enable reading measurement results, measuring may be paused with the STOP button and started with the START button.

5.5. Test results

Maximum absolute percent errors of frequency measurement, $\max |\delta_f|$, and amplitude measurement, $\max |\delta_A|$, are shown Tables 1 and 2.

Table 1. Measurement Errors in the Measuring System with DSP for 60 Samples of the Measured Signal

Test No.	$\max \delta_f $	$\max \delta_A $
1	0.11%	0.32%
2 (mod. 20%)	0.55%	2.2%
2 (mod. 10%)	0.42%	1.5%
2 (mod. 5%)	0.26%	1.1%
2 (mod. 3%)	0.17%	0.8%
6	0.56%	16.17%
7	0.50%	20%

Table 2. Measurement errors got in simulation with Matlab for 60 and 100 samples of the measured signal

Test No.	$\max \delta_f $		$\max \delta_A $	
1	1e-4%	4e-5%	0.013%	0.003%
2	0.17%	0.06%	0.09%	0.04%
3	0.53%	0.26%	3.00%	2.45%
4	0.50%	0.23%	2.70%	1.95%
6	0.60%	0.22%	20.00%	20.00%
No. of samples	60	100	60	100
Measuring time	0.7 sec	1.2 sec	0.7 sec	1.2 sec

6. CONCLUSIONS

The task was performed according to the assumptions made. The advantage of the presented solution lies in that the uncertainty of frequency measurement of the distorted signal is 0.23% for measuring time of about 1.2 sec or 0.53% for measuring time of about 0.7 sec.

The superiority of this method over others consists in that attaining such accuracy of frequency measurement by other methods requires longer measuring time and its limitation to about 1 sec leads to a substantial reduction of measurement accuracy, greater than that in the described method. Moreover, signal sampling time is shorter than 1 second, which means that, using a faster signal processor, measurement time can be shortened substantially with no accuracy reduction.

The measurement speed related to the rate of changes of measured parameters is sufficient, and, thanks to oscillation reduction, improving of accuracy does not cause a proportional extension of measurement time.

The presented results were obtained during experiments in which some portions of the MS thesis of Paweł Pachota and Robert Mocio, supervised by Dr. Andrzej Bień, were used.

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