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NUMERICAL DETERMINING ELECTRICAL RESISTANCE AND REACTANCE OF TRANSMISSION LINE

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Summary: **The paper deals with a numerical method for evaluation of electrical resistance and reactance of straight parallel conductors with different shapes of cross section - the method described by Enrico Comellini, Angelo Invernizzi, Giancarlo Manzoni [1] and simplified by Béla Török and György Varju [2]. This method is called as The Method of Elementary Conductors and we have used the method for calculation of impedance of leads transmitting electrical energy from transformer to electric-arc furnace (so called short circuit). Our goal in this article is to describe the most suitable mathematical model (from point of accuracy and calculation speed) that we found during our work. The mathematical model was created in programming environment MATLAB.**

1. INTRODUCTION

Accurate calculation of the electrical resistance and inductance of conductors is very important especially for leads of low-ohmic appliances (e.g. electric-arc furnaces). Impedance of leads and appliances is comparable and parameters and qualities of the leads therefore strongly influence characteristics of the whole device. There are many mathematical and experimental methods for evaluation of the impedance. From mathematical methods we dealt with some analytical solutions and two numerical methods—the finite difference method and the method of elementary conductors. The analytical solutions were usually very accurate and suitable for programming but they were valid only for special types and configurations of conductors. The numerical methods were much more universal but not as accurate as analytical ones and calculation took a longer time. Experimentally we have measured the conductor parameters by calorimetric method but it was very difficult to keep conditions for accurate measurement and to obtain the results took much more time than by calculation. It should be emphasised that apparatus for measuring is incomparable more expensive than tools for computing. So from practical viewpoint and viewpoint of universality it was suitable to focus attention to mathematical methods especially to numerical ones. From the two numerical methods mentioned above the method of elementary conductors was chosen as more appropriate because of its better stability and accuracy.

2. METHOD OF ELEMENTARY CONDUCTORS

Method of elementary conductors is a numerical method based on dividing the conductor with non-uniform current density in cross section into smaller conductors. Constant value of current density is assumed in cross section of these smaller conductors. Dimensions of cross section of the conductors must be smaller than the skin depth in order to respect real current distribution (Fig.1). The skin depth is given by formula:

$$a = \sqrt{\frac{2}{\omega\mu\gamma}} \tag{1}$$

where a is the skin depth, ω is angular frequency, μ is permeability of the conductor and γ is its conductivity.

The smaller conductors are called the elementary conductors. It is possible to evaluate resistance of elementary conductors and their self and mutual inductance. In this way, there is obtained series-parallel *R-L* circuit in which elementary currents $\dot{I}_e$ can be calculated (Fig.2—conductors marked 1.,2.,.....,N are divided by elementary conductors that are supplied by sources *Z*1, *Z*2,....., *Zn* and circuit is closed by reference conductor with resistance which represents ground). Elementary currents characterise current distribution in cross section of the conductors so it is possible to calculate their impedance per unit length ***Z*** as:

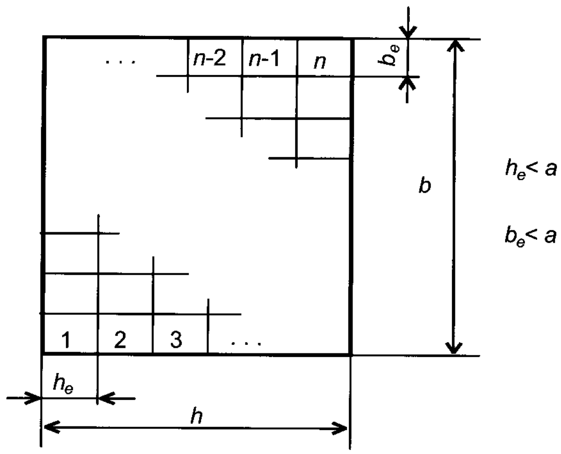

Fig.1 Dividing of Cross Section of Conductor by Elementary Conductors

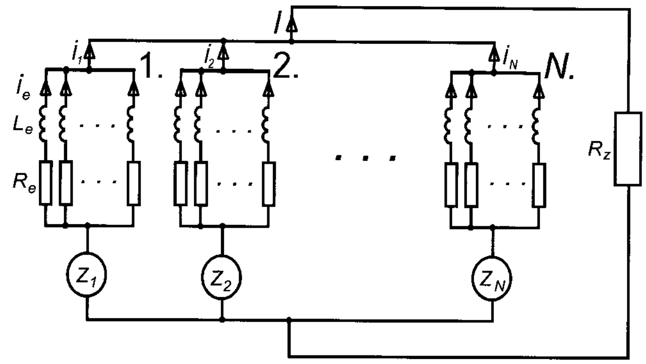

Fig.2. Series-parallel *R-L* Circuit of System of Elementary Conductors

$$\boldsymbol{Z}_i = \frac{\boldsymbol{U}_i - R_z \cdot \boldsymbol{I}}{\boldsymbol{I}_i} \qquad (2)$$

where $\boldsymbol{Z}_i$ is impedance of the *i*-th conductor per unit length, $\boldsymbol{U}_i$ is voltage of source supplying the *i*-th conductor, $\boldsymbol{I}_i$ is current flowing through the *i*-th conductor (given as the sum of all elementary currents in *i*-th conductor), R_z is resistance of the reference conductor per unit length, $\boldsymbol{I}$ is current flowing through the reference conductor (given as the sum of all currents flowing through conductors).

When the impedance is known, it is easy to evaluate resistances and inductivities of the lead conductors or coefficients of the skin and proximity effect.

3. INFLUENCE OF METHOD OF DIVIDING THE CONDUCTOR CROSS SECTION ON ACCURACY

The way of dividing conductor cross section to elementary conductors influences way of computing the electrical parameters so influences the computation accuracy. Electrical parameters of the elementary conductors and resistance R_z are given by formulas:

— resistance R_z per unit length:

$$R_z = 9.9 \cdot 10^{-7} \cdot f \qquad (3)$$

where f is current frequency

— resistance of elementary conductor per unit length:

$$R_e = \rho \frac{1}{S_e} \qquad (4)$$

where R_e is resistance of elementary conductor, ρ is conductor resistivity and S_e is cross section area of elementary conductor

— self and mutual inductance of elementary conductor per unit length:

$$L_{ij} = 2 \cdot 10^{-7} \ln \frac{D_z}{F_{ij}} \qquad (5)$$

where L_{ij} is mutual (when $i=j$ self) inductance, D_z is so called diameter of reference tube and F_{ij} is mutual (when $i=j$ self) average geometrical distance.

For D_z are valid relations:

$$D_z = 659\sqrt{\frac{\rho_z}{f}}, \quad D_z \geq 10 \cdot D_{ij\,\max} \qquad (6)$$

where ρ_z is resistivity of ground (5–100 Ωm) and $D_{ij\,\max}$ is maximum distance between elementary conductors in space.

F_{ij} can be evaluated from equation:

$$\ln F_{ij} = \frac{1}{h_{ei} \cdot b_{ei} \cdot h_{ej} \cdot b_{ej}} \cdot \frac{1}{2} \int_{x=-\frac{h_{ei}}{2}}^{\frac{h_{ei}}{2}} \int_{y=-\frac{b_{ei}}{2}}^{\frac{b_{ei}}{2}} \int_{\xi=c-\frac{h_{ej}}{2}}^{c+\frac{h_{ej}}{2}} \int_{\eta=d-\frac{b_{ej}}{2}}^{d+\frac{b_{ej}}{2}} \ln\left[\frac{(x-\xi)^2+(y-\eta)^2}{l^2}\right] dx\,dy\,d\xi\,d\eta \qquad (7)$$

meaning of symbols used in describes Fig.3.

It is clear that evaluation of average geometrical distances by formula (7) is much too difficult but for some shapes of conductor cross section simplified equations there are derived:

— rectangular cross section

$$F_{ii} \approx 0.2234(h_e + b_e), \quad F_{ij} \approx 1.0065\sqrt{X_{ij}^2 + Y_{ij}^2} \qquad (8)$$

where h_e (b_e) is width (thickness) of elementary conductor cross section and X_{ij} (Y_{ij}) is centre distance of elementary conductors in direction of axis x (y)

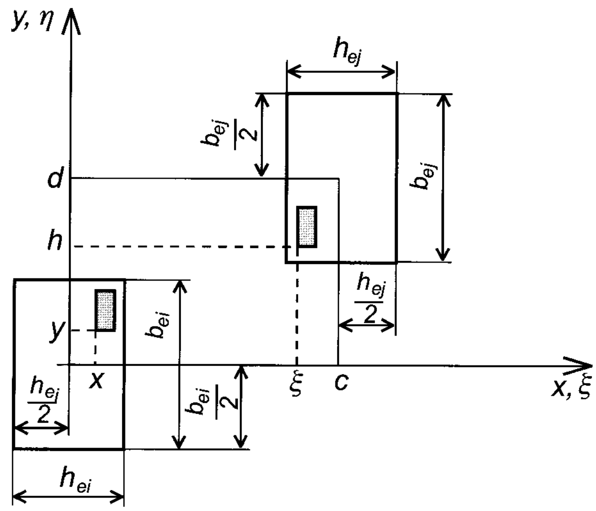

Fig. 3. Geometrical Arrangement of Elementary Conductors

$$F_{ii} \approx \sqrt[16]{\left[0.2234\left(h_{ei}+\overline{TS}_i\frac{\pi}{2n_\varphi}\right)\right]^4\left(2\overline{TS}_i\sin\frac{\pi}{4n_\varphi}\right)^2 \ldots\ldots}$$

$$\overline{\ldots\ldots\left(2\overline{TS}_i\sin\frac{\pi}{2n_\varphi}\right)^2\left(2\overline{TS}_i\sin\frac{3\pi}{4n_\varphi}\right)^2\left(2\overline{TS}_i\sin\frac{\pi}{4n_\varphi}\right)^2 \ldots\ldots}$$

$$\overline{\ldots\ldots\left(2\overline{TS}_i\sin\frac{\pi}{2n_\varphi}\right)^2\left(2\overline{TS}_i\sin\frac{\pi}{4n_\varphi}\right)^2} \tag{10}$$

$$F_{ij} \approx \sqrt{X_{ij}^{\ 2}+Y_{ij}^{\ 2}}$$

— circular cross section

$$F_{ii} \approx 0.778\frac{d_e}{2},\ \ F_{ij} \approx \sqrt{X_{ij}^{\ 2}+Y_{ij}^{\ 2}} \tag{9}$$

where d_e is diameter of elementary conductor cross section. So F_{ij} and then L_{ij} calculation depends on a shape of the elementary conductor cross section, hence we focused our attention to finding the best way of dividing conductors into elementary ones.

3.1. Dividing the conductors with circular cross section

It was obvious to divide conductors with circular cross section by elementary conductors with segment of annulus cross section. Every dimension of the annulus segment was smaller than the skin depth. For average geometrical distances are valid simplified relations:

where n_φ is number of elementary conductors in direction of axis φ, h_{ei} is thickness of the i-th elementary conductor and $\overline{TS}_i$ is the distance between centre of cross section of elementary conductor and centre of cross section of divided conductor. Accuracy of calculation was not very good in this case because the error of calculated values (obtained by comparison of the evaluated values with data published in literature) was up to 7.5%. Therefore we tried to improve the accuracy by substitution of the elementary conductors with segment of annular cross section by elementary conductors with circular cross section, Fig.4. There in this case equations (9) were used for average geometrical distances. Maximum error of section by elementary conductors with circular cross section calculated values decreased to 5.4% thus the substitution can be considered as suitable. Even better results were obtained by substitution of the elementary conductors with segment of annular cross section by elementary conductors with square cross section (Fig.5) for which were used formulas (8). In this case maximum error decreased to 4.7% but we wanted to improve calculation accuracy more.

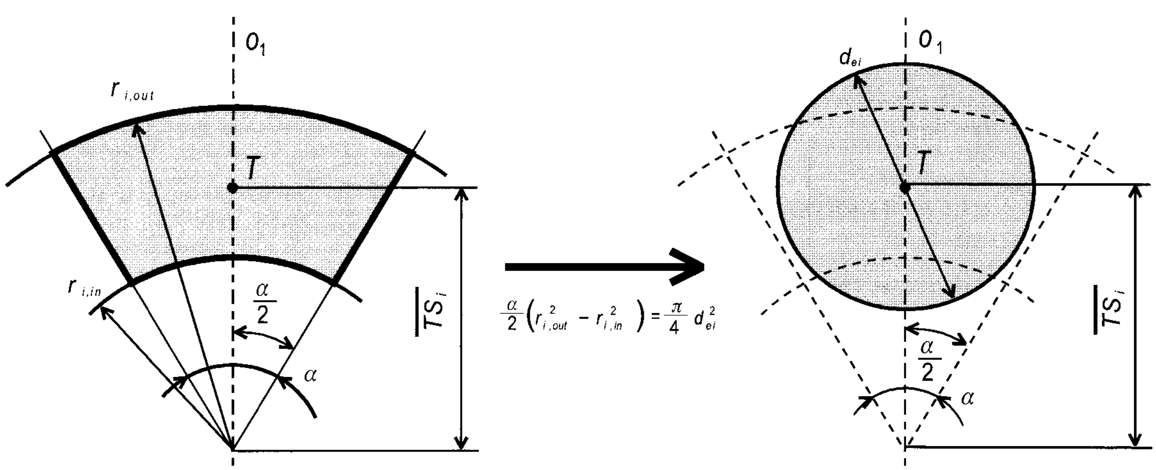

Fig. 4. Substitution of the Elementary Conductors with Segment of Annular Cross Section by Elementary Conductors with Circular Cross Section

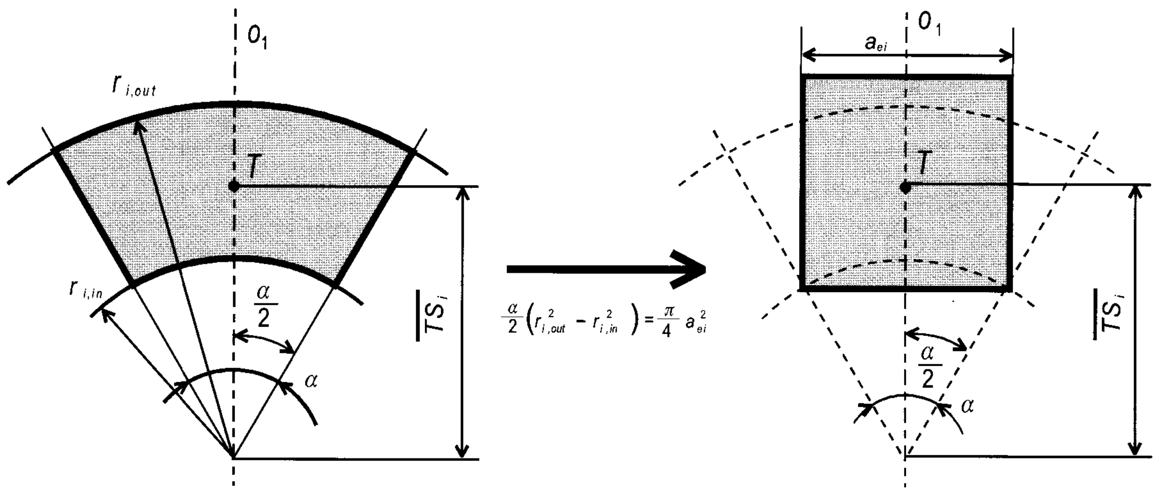

Fig. 5. Substitution of the Elementary Conductors with Segment of Annular Cross Section by Elementary Conductors with Square Cross Section

So we tried to increase the number of elementary conductors by reducing their cross section dimensions. For dimensions smaller than half of the skin depth the error was smaller than 1.4% and calculation speed was not significantly influenced. Additional increasing of the number of elementary conductors resulted in more accurate calculation but the speed of evaluation decreased even more.

3.2. Dividing the conductors with rectangular cross section

In this case there was obvious to divide the conductor cross section by elementary conductors with rectangular cross section, too. When dimensions of elementary conductor cross section were smaller than the skin depth the error of obtained results was up to 5.9%. This error did not decrease by substitution of the elementary conductor with rectangular cross section by the elementary conductors with circular shape but it increased to 7%. So the only way to get better accuracy was to increase the number of elementary conductors. By doubling their error fell to 4.3% but calculation took much more time. Therefore additional increasing of elementary conductors number was not reasonable. The error greater than that for conductors with the circular cross section was caused by larger cross section areas and by more complicated current distribution.

4. CONCLUSIONS

Basing on our experience we can state that the Method of Elementary Conductors, due to its simplicity, universality, stability and accuracy, is very suitable for evaluation of resistance and inductance of straight conductors. The method is also appropriate for computer programming.

To get as accurate results as possible we recommend for the conductors with circular cross section to substitute elementary conductors with segment of annular cross section by the elementary conductors with square cross section and to double the number of the elementary conductors in every axis direction.

In order to improve the calculation accuracy for rectangular conductors it is necessary to increase the number of elementary conductors more than twice in every axis direction although it causes significant reduction of the computation speed. It is however clear that in the next few years this problem will not be topical because of rapid development of computer technologies.

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