

SUPPLY NETWORK CURRENT SYMMETRIZATION—A GRAPHICAL APPROACH

Aleksy KURBIEL

Zbigniew HANZELKA

University of Mining and Metallurgy
Kraków (Poland)

Summary: The three methods of selecting elements for symmetrising network currents and at the same time compensating the reactive power are presented. To select the symmetrising elements, it is necessary to measure the phase voltages U_{fi} ($i = 1, 2, 3$) of the network, line currents I_i , phase active power P_i and reactive power Q_i of the network.

1. INTRODUCTION

High power loads such as arc furnaces in steel-making often absorb asymmetrical currents from the network. Too large asymmetry of the currents is unfavorable. In such cases, the necessity of equalizing the currents where the industrial plant absorbs its power could arise.

Unequal currents can be symmetrised by connecting to the network the branch with the relevant electrical elements.

A load generating asymmetrical currents in a network can be replaced with two equivalent loads: a symmetrical three-phase and single-phase loads which can be symmetrised by means of reactors and capacitors to equalize the currents within the supply network.

Also, it can be assumed that the current asymmetry of the supply network is a result of three, delta-connected, single-phase loads of different impedances. After the symmetrisation each of them, by means of reactors and capacitors, the current equalization of the supply network will be completed.

The network current can also be symmetrised by connecting to the network three delta-connected capacitors of different capacitances.

The above-mentioned methods of current network symmetrisation resulting in the simultaneous equalization of active and reactive phase powers of the network are analysed below. The problem of current symmetrisation is strictly connected with the problem of compensation of the network reactive power, so both these problems will be discussed together. The following assumption have been made:

- there is no neutral wire in the three-phase network;
- the network load is of a resistive-inductive character;
- the phase-to-phase voltages U_{pi} and phase voltages U_{fi} of the network are symmetrical;

- the angles j_i between the respective network currents I_i and phase voltages U_{fi} ($i = 1, 2, 3$) are known. The angles j_i can be calculated using the measured voltages U_{fi} , currents I_i and phase powers P_{fi} of the network.

2. DETERMINATION OF THE SINGLE-PHASE CURRENT EQUALIZING THE NETWORK CURRENTS

A method of symmetrising the unequal network currents: $I_1 \neq I_2 \neq I_3$ by means of the single-phase system of adequate elements which generates the required symmetrising current, is presented in Fig. 1. Symmetric currents of the supply network: I_1 , $I_{21} = a^2 I_1$, $I_{31} = a I_1$ ($a = -0.5 + j\sqrt{3}/2$) occur when a system generating the current I_{d2} and satisfying the equation $I_2 + I_{d2} = I_{21}$ is connected to the voltage U_{p2} .

The current I_{d2} increasing the current I_3 to the value of I_{31} will occur in the third phase.

This results from the equation: $I_1 + I_2 + I_{d2} + I_3 - I_{d2} = 0$ and $I_1 + I_2 + I_3 = 0$. The current I_{d2} which symmetrizes the network currents, is delayed by a small angle relative to the voltage U_{p2} . To generate such a current, it is necessary to connect to the voltage U_{p2} the branch with the resistance R_{d2} and a relatively small reactance X_{d2} . An additional power loss: $R_{d2} I_{d2}^2$ will occur in this branch.

Another symmetry of network currents I_2 , $I_{32} = a^2 I_2$, $I_{12} = a I_2$ will occur after connecting to the voltage U_{p3} the branch where the current I_{d3} satisfying the equation $I_3 + I_{d3} = I_{32}$ will occur. The current I_{d3} is delayed by an angle greater than 120° relative to the voltage U_{p1} . Such a current will occur in the branch with the negative resistance, i.e. a power source, and the inductive reactance.

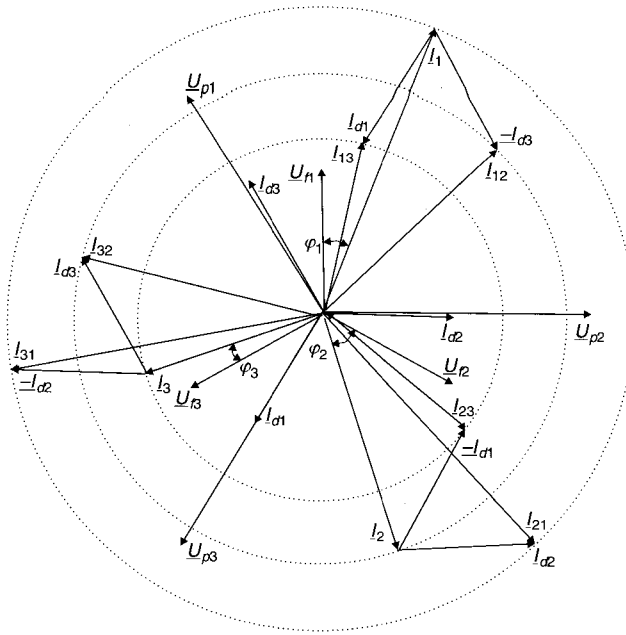


Fig. 1. Graphical determination of the symmetrising currents I_{di}

Another symmetry of the network currents I_3 , $I_{13} = a^2 I_3$, $I_{23} = a I_3$ can be obtained by means of the branch connected to the voltage U_{p1} where the current $I_{d1} = I_{13} - I_1$ with the phase angle of almost 120° ahead of the phase voltage U_{p1} will flow. The branch should therefore include an adequate power source and capacitors.

The three different systems presented of current symmetrisation that can be made of branches with the adequate symmetrising elements are unsuitable for industrial use for economic and technical reasons (due to greater power loss or sometimes the power source required).

3. THE SINGLE-PHASE LOAD SYMMETRISATION SYSTEM

The loads charging the network with asymmetrical currents can be replaced by an equivalent system consisting of a symmetrical three-phase load and the branch with adequate elements. This is explained in Fig. 2 which differs from Fig. 1 in the opposite senses of currents I_{di} .

According to the following equations, the system of asymmetrical network currents $I_1 \neq I_2 \neq I_3$ can be replaced by one of the three different symmetrical currents and a single-phase current:

$$\begin{aligned} \begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix} &= \begin{bmatrix} a^2 I_3 + I_{d1} & a I_3 - I_{d1} & I_3 \end{bmatrix} \\ &= \begin{bmatrix} a I_2 - I_{d3} & I_2 & a^2 I_2 + I_{d3} \end{bmatrix} \\ &= \begin{bmatrix} I_1 & a^2 I_1 + I_{d2} & a I_1 - I_{d2} \end{bmatrix} \end{aligned} \quad (1)$$

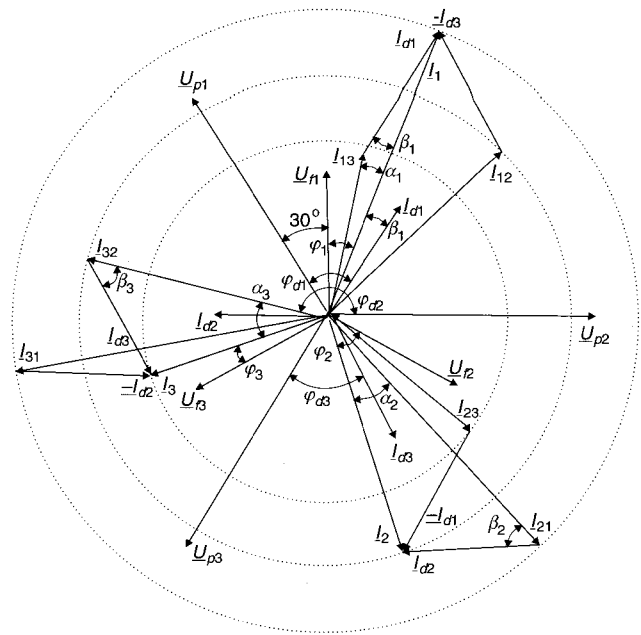


Fig. 2. Vector diagram of the currents and voltages described by equations (1)

It can be assumed for the first of the above-mentioned equations, that two loads: namely three-phase and single-phase are connected to the network. The three-phase load absorbs equal currents from the network: $I_{13} = I_{23} = I_3$. The active and reactive powers of this load are:

$$P_3 = 3U_f I_3 \cos \varphi_3; \quad Q_3 = 3U_f I_3 \sin \varphi_3 \quad (2)$$

The single-phase load connected to the voltage U_{p1} absorbs the current I_{d1} which can be determined by the equation:

$$I_{d1} = \sqrt{I_1^2 + I_{13}^2 - 2I_1 I_{13} \cos \alpha_1}; \quad \alpha_1 = \varphi_1 - \varphi_3 \quad (3)$$

Compared to the voltage U_{p1} , this current is delayed by the angle $\varphi_{d1} = 30^\circ + \varphi_1 + \beta_1 < 90^\circ$, where:

$$\sin \beta_1 = \frac{I_{13}}{I_{d1}} \sin \alpha_1 \quad (4)$$

The single-phase load representing asymmetry is a series connection of the resistance R_{d1} and inductive reactance X_{Ld1} equal to:

$$R_{d1} = \frac{U_p}{I_{d1}} \cos \varphi_{d1}; \quad X_{Ld1} = \frac{U_p}{I_{d1}} \sin \varphi_{d1} \quad (5)$$

In the relevant professional literature, e.g. [2], symmetrisation systems for a single-phase load are described. A resistive-inductive load meeting the condition:

$$\operatorname{tg} \varphi_{d1} = \frac{X_{Ld1}}{R_{d1}} \left(-\frac{1}{\sqrt{3}} \right) \quad (6)$$

constitutes a symmetrical system with a capacitor and reactor of reactances:

$$X_{C2} = \frac{R_{d1}^2 + X_{Ld1}^2}{\frac{R_{d1}}{\sqrt{3}} - X_{Ld1}}; \quad X_{L3} = \frac{R_{d1}^2 + X_{Ld1}^2}{\frac{R_{d1}}{\sqrt{3}} - X_{Ld1}} \quad (7)$$

connected to the voltages U_{p1} and U_{p2} , respectively (Fig. 3). Replacing the phases of the networks connected to either capacitors or a reactor results in the desymmetrisation of the system.

For a load for which the equation $X_{Ld1} = R_{d1} / \sqrt{3}$ holds, a symmetrising element is a reactor only of the reactance $X'_{L3} = 2X_{Ld1}$ ($X_{C2} \rightarrow \infty$). If the inequality $\operatorname{tg} \varphi_{d1} = X_{d1} / R_{d1} > 1/\sqrt{3}$ holds, then instead of the capacitor C_2 in the system as in Fig. 3, a second reactor of the reactance:

$$X'_{L2} = \frac{R_{d1}^2 + X_{d1}^2}{X_{d1} - \frac{R_{d1}}{\sqrt{3}}} \quad (8)$$

should be used to obtain the symmetry of the line currents.

Symmetrisation of single-phase load currents can be also achieved in such a way that the whole reactive power ($Q_{d1} = X_{Ld1} I_{d1}^2$) of the load is first compensated for by a capacitor (connected parallel to the load) of reactance:

$$X_{CK} = \frac{R_{d1}^2 + X_{Ld1}^2}{X_{Ld1}} \quad (9)$$

and then, according to Fig. 3, a symmetrising capacitor and reactor of the equivalent reactances:

$$X'_{C2} = X'_{L3} = \sqrt{3} R_{Z1} \quad (10)$$

are connected to the load with a capacitor of the equivalent resistance:

$$R_{Z1} = \frac{R_{d1}}{1 - \frac{X_{Ld1}}{X_{CK}}} \quad (11)$$

The symmetric system of a single-phase load with capacitors and reactor absorbs only the active power $P_{d1} = R_{d1} I_{d1}^2$ and loads the network with symmetrical currents $I_{ui} = P_{d1} / (3U_f)$ being in phase with the phase voltages \underline{U}_{fi} ($i = 1, 2, 3$).

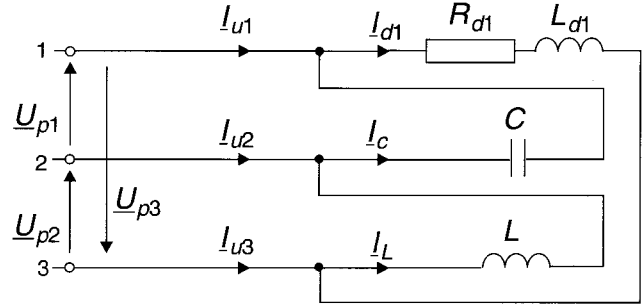


Fig. 3. The symmetrisation system of a single-phase load: $R_{d1} + jX_{d1}$

The network is loaded with the total active power $P_s = P_3 + P_{d1}$ and reactive power Q_3 (eq. 2). If $\operatorname{tg} \varphi_s = Q_3 / P_s$ is too large, then the reactive power Q_3 may be decreased by such a quantity Q'_3 so that $\operatorname{tg} \varphi_s = (Q_3 - Q'_3) / P_s$ would not exceed the level at which the fee for the reactive power is charged. The reactive power will decrease by Q'_3 when identical capacitors of reactance $X'_{CK} = 3U_p^2 / Q'_3$ are connected to each network phase-to-phase voltage. As a consequence of the current symmetrisation and the compensation of the reactive power, two capacitive reactances X_{CK} (eq. 9) and X'_{CK} that correspond with the two capacitances C_K and C'_K are connected to the voltage U_{p1} , so the equivalent capacity of the two group of capacitors is $C_{Z1} = C_K + C'_K$. Two capacitors with a capacity of $C_{Z2} = C'_2 + C'_K$ are connected to the voltage U_{p2} where C'_2 results from the symmetrisation reactance X'_{C2} .

The inductive reactance X'_{L3} (11) and capacitive reactance X'_{CK} are connected to the voltage U_{p3} . Instead of these reactances, the equivalent reactance X_{Z3} determined from the relationship $(X_{Z3})^{-1} = (X'_{CK})^{-1} - (X'_{L3})^{-1}$ should be connected. If $(1/X_{Z3}) > 0$ then the capacitors of capacity $C_{Z3} = 1/(\omega X_{Z3})$ are the equivalent reactance; if $(1/X_{Z3}) < 0$ then X_{Z3} is the reactor reactance.

The second equality (1) means that the given unequal network current I_i will occur when we connect to the network both: a three-phase symmetric load with the currents $I_{12} = I_2 = I_{32}$, the powers $P_2 = 3U_f I_2 \cos \varphi_2$ and $Q_2 = 3U_f I_2 \sin \varphi_2$, and an additional single-phase symmetric load connected to the voltage U_{p3} absorbing the current I_{d3} shifted ahead of the voltage by the angle of $\varphi_{d3} = 150^\circ - \alpha_3 - \beta_3 - \varphi_3 < 90^\circ$. The load is of resistive-capacitive character.

The current I_{d3} , angles α_3 and β_3 (Fig. 2), the resistance R_{d3} and the capacitive reactance X_{Cd3} of the single-phase load can be calculated using relationships similar to (3-5). The active power of the load is $P_{d3} = R_{d3} I_{d3}^2$ and the capacitive reactive power $Q_{Cd3} = X_{Cd3} I_{d3}^2$. If $X_{Cd3} < R_{d3} / \sqrt{3}$ then the capacitor and reactor of reactances:

$$X_{C1} = \frac{R_{d3}^2 + X_{Cd3}^2}{\frac{R_{d3}}{\sqrt{3}} + X_{Cd3}}; \quad X_{L2} = \frac{R_{d3}^2 + X_{Cd3}^2}{\frac{R_{d3}}{\sqrt{3}} - X_{Cd3}} \quad (12)$$

are the symmetrising elements, where the capacitor should be connected to the voltage U_{p1} and the reactor to the voltage U_{p2} , as in Fig. 3. If $X_{Cd3} = R_{d3} / \sqrt{3}$ then the only sym-

of the voltage U_p , currents I_{0i} and power P_{0i} enables the current transformers to determine their resistances R_{0i} and reactances X_{0i} . The reactive power $Q_0 = X_{0i} I_{0i}^2$ of each load can be compensated for by the capacitors connected in parallel to the loads with reactances:

$$X_{CKi} = \frac{U_p^2}{Q_{0i}} = \frac{R_{0i}^2 + X_{0i}^2}{X_{0i}} \quad (15)$$

where the equivalent resistance R_{Zi} of a load with the capacitor is:

$$R_{Zi} = \frac{R_{0i}}{1 - \frac{X_{0i}}{X_{CKi}}} \quad (16)$$

The resistances R_{Zi} together with the capacitive reactances X_{CSi} and inductive X_{LSi} which satisfy the equations:

$$\begin{aligned} X_{CS2} &= X_{LS3} = \sqrt{3}R_{Z1}; & X_{CS3} &= X_{LS1} = \sqrt{3}R_{Z2}; \\ X_{CS1} &= X_{LS2} = \sqrt{3}R_{Z3} \end{aligned} \quad (17)$$

and are connected to the respective voltages U_i of the network, constitute symmetrical systems. The subscripts of the reactances detail which line voltages they are to be connected to. Thus the reactances X_{CK1} , X_{CS1} , X_{LS1} are connected to the voltage U_{p1} and their equivalent value X_{Z1} can be described by the expression:

$$\frac{1}{X_{Z12}} = \frac{1}{X_{CK1}} + \frac{1}{X_{CS1}} - \frac{1}{X_{LS1}}.$$

Taking into account the equality of $X_{LS1} = X_{CS3}$, this expression can be written as:

$$\frac{1}{X_{Z1}} = \omega C_{Z1} = \omega(C_{K1} + C_{S1} - C_{S3}) \quad (18a)$$

The equivalent reactances X_{Z2} and X_{Z3} are connected to the voltages U_{p2} and U_{p3} where the inversions of the reactances are:

$$\frac{1}{X_{Z2}} = \omega C_{Z2} = \omega(C_{K2} + C_{S2} - C_{S1}) \quad (18b)$$

$$\frac{1}{X_{Z3}} = \omega C_{Z3} = \omega(C_{K3} + C_{S3} - C_{S2}) \quad (18c)$$

Connecting the capacitors of capacitance C_{Z1} , C_{Z2} , C_{Z3} into the network results in symmetrical currents being in phase with the respective phase voltages U_{f1} .

The network power factor can be reduced (to $\cos \varphi_s < 1$) by disconnecting a part of the capacitors with identical capacitances from each voltage.

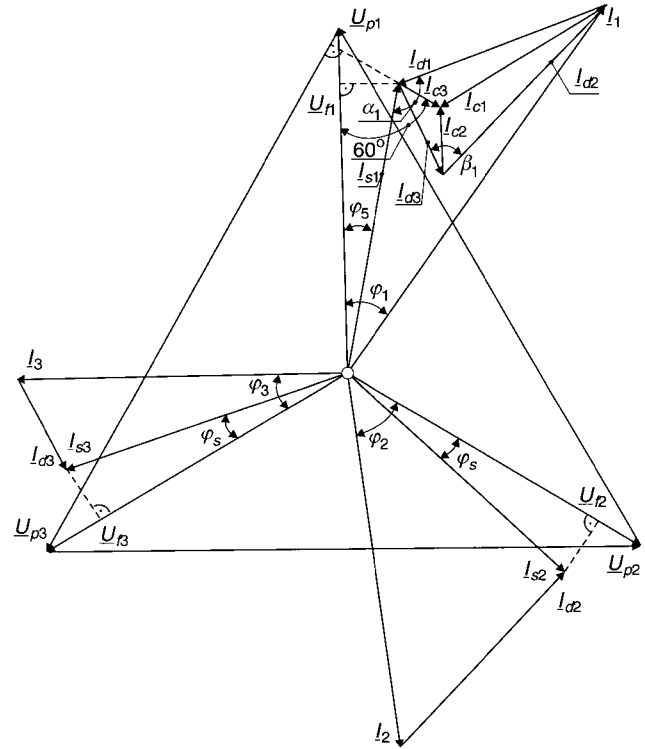


Fig. 5. Graphical determination of currents I_{d3} , I_{C2} , I_{C3} of the symmetrising capacitors

4. DELTA SYSTEM OF SYMMETRISATION CAPACITORS

A detailed analysis of a system symmetrising the network load is included in [1]. A graphical method for determination of the currents of the delta-connected capacitors is presented in Fig. 5, where they both equalize the network currents and maintain the preset value of the power factor.

The active and reactive powers, P and Q of an asymmetrical load are respectively:

$$\begin{aligned} P &= U_f (I_1 \cos \varphi_1 + I_2 \cos \varphi_2 + I_3 \cos \varphi_3) \\ Q &= U_f (I_1 \sin \varphi_1 + I_2 \sin \varphi_2 + I_3 \sin \varphi_3) \end{aligned} \quad (19)$$

Identical active power $P_S = P$ occur at symmetrical currents I_S whose active components satisfy the equation:

$$I_S \cos \varphi_3 = \frac{1}{3} (I_1 \cos \varphi_1 + I_2 \cos \varphi_2 + I_3 \cos \varphi_3) \quad (20)$$

Such a value of $\tan \varphi_S = Q_S / P_S < Q / P$ can be adopted so that no fee is charged for the reactive power. The reactive power Q_S occurring at symmetrical currents I_{Si} is equal: $Q_S = 3U_f I_S \sin \varphi_S$.

To obtain the symmetrical currents I_{Si} , additional currents I_{di} should be added to the currents I_i . The current I_{d1} is determined by the relationship:

$$I_{d1} = \sqrt{I_1^2 + I_{S1}^2 - 2I_1I_{S1} \cos(\varphi_1 - \varphi_S)} \quad (21)$$

The currents I_{d2} and I_{d3} can be calculated using similar relationships.

Because $I_1 + I_2 + I_3 = 0$, then the sum of the currents $I_{d1} + I_{d2} + I_{d3} = 0$. In consequence the line currents I_{di} form a delta (Fig. 5). The angle β_1 of this delta-form opposite to the side I_{d1} can be calculated using the relationship:

$$\operatorname{tg} \frac{\beta_1}{2} = \sqrt{\frac{(p - I_{d2})(p - I_{d3})}{p(p - I_{d1})}} \quad \text{where}$$

$$p = \frac{1}{2}(I_{d1} + I_{d2} + I_{d3}) \quad (22)$$

Likewise the angle β_2 opposite to the side I_{d2} and the angle β_3 opposite to the side I_{d3} can be calculated.

The currents I_{Ci} are added to the delta of line currents in such a way that $I_{C1} \perp U_{p1}$, $I_{C2} \perp U_{p2}$, $I_{C3} \perp U_{p3}$, i.e. the angles between these currents are 120° . The line currents constitute the differences of the respective phase currents: $I_{d1} = I_{C1} - I_{C3}$, $I_{d2} = I_{C2} - I_{C1}$, $I_{d3} = I_{C3} - I_{C2}$. The currents I_{C1} and I_{C3} can be calculated from the current delta: I_{C1} , I_{C3} , I_{d1} . In this delta the angle $\gamma_1 = \angle(I_{d1}, I_{C3}) = \alpha_1 + \varphi_S - 60^\circ$ and the angle $\gamma_3 = \angle(I_{d1}, I_{C1}) = 60^\circ - \gamma_1$. The currents I_{C1} , I_{C3} can be determined knowing the current I_{d1} and the angles γ_1 and γ_3 . A knowledge of the currents I_{d3} and I_{C3} and the angle $\gamma_2 = \angle(I_{d3}, I_{C3}) = \beta_2 - \gamma_3$ ($\beta_2 = \angle(I_{d1}, I_{d3})$) enables the current I_{C2} to be calculated.

In order to have symmetrical currents in the network, of I_s and, at the same time, shifted in phase by the angle φ_S vs the respective phase voltages U_{fi} of the network, a capacitor of reactance $X_{C1} = U_p / I_{C1}$ should be connected to the voltage U_{p1} , a capacitor of reactance $X_{C2} = U_p / I_{C2}$ should be connected to the voltage U_{p2} , and a capacitor of reactance $X_{C3} = U_p / I_{C3}$ should be connected to the voltage U_{p3} . After symmetrisation of the currents in the network to I_s , the reactive power Q_S will be less than the power Q (eq. 19). The difference of these powers is equal to the power of the symmetrisation capacitors of reactances X_{Ci} , i.e.:

$$Q - Q_S = U_p^2 \left(\frac{1}{X_{C1}} + \frac{1}{X_{C2}} + \frac{1}{X_{C3}} \right) \quad (23)$$

5. CONCLUSIONS

The method of symmetrisation of network currents, given as the first, is not suitable for practical purposes due to the large additional loss of active power.

The three subsequent methods of selecting elements for symmetrising network currents and at the same time compensating the reactive power are equivalent because the elements calculated by each of these methods enables the same network load level to be achieved.

To select the symmetrising elements, it is necessary to measure the symmetrical voltages U_{fi} ($i=1, 2, 3$), line currents I_i , phase active power P_i and reactive power Q_i of the network.

When the calculations given in Section 3 are employed, an additional system for measuring the phase currents I_{fi} (Fig. 4) is needed. The measurement results enables the symmetrisation currents and, based on this, the symmetrising elements to be calculated.

6. REFERENCES

1. Hanzelka Z.: *Efficiency of static compensation applied to reduction of the variable loads influence on supply network* (in Polish), Wydawnictwa AGH, Kraków 1994.
2. Kurbiel A.: *Inductive electrothermal devices* (in Polish), Wydawnictwa AGH, Kraków 1992.

Prof. Aleksy Kurbiel

Mailing address:
Ul. Obopólna 3/24, 30-069 Cracow
POLAND



Zbigniew Hanzelka

Associate Professor in the Institut of Electrical Drive and Industrial Equipment Control of UMM. His areas of interest include electrical power quality, particularly methods of reducing the influence of power converters on supply network. He is the member of several national and international committees, among others in IEC 77A, UIE, CIGRE.

Mailing address:
University of Mining and Metallurgy
30-059 Cracow, Al. Mickiewicza 30, POLAND
phone: +48-12-617 28 78
fax: +48-12-633 22 84
e-mail: hanzel@uci.agh.edu.pl