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A METHOD TO SEPARATE LOAD-SIDE AND SUPPLY-SIDE HARMONICS

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Summary: **The contribution of a single customer to the harmonic distortion of the supply voltage is a difficult subject to deal with. This paper describes a method to separate the individual harmonic voltage components caused by the supply-side and load-side using basic linear circuit theory. Both components are highly dependent on the pre-existing quality of the source voltage, the equivalent source impedance, load impedance and load harmonic current.**

1. INTRODUCTION

It is well known that present power distribution systems carry some level of harmonics in the voltage waveform due to nonlinearity of some customer loads. As more power electronics applications find their way into the market, voltage distortion will continue to rise, and current recommended limits [1] will likely be enforced.

With the exception of the nonsinusoidal excitation current of power transformers, the voltage distortion found in the utility supply is caused customer nonlinear loads. Literally, part of the load of each individual customer operates with nonsinusoidal current. Nonlinear loads distort the voltage supply in a complex manner as the system reacts to each harmonic in a different way: The network's circuit configuration (capacitors, inductors and resistors) has a nonlinear frequency response and harmonic voltage due to a fixed harmonic current may be several times larger at a some frequencies than others. As a consequence, the amount of harmonics contributed by each individual customer depends not only on the size of his or her nonlinear load, but also on the linear load component, the equivalent harmonic source impedance and pre-existing harmonic voltage at the customer point.

Meanwhile, the contribution of a customer to the waveform distortion is of growing interest to the utility industry. Some proposed methods to recover the cost of harmonic pollution from those end-users that cause disturbances have already began to emerge [2, 3]. The question is how much harmonic distortion is attributed to a specific customer and how much is attributed to the equivalent power supply (utility and other customers combined). Attempts have been made to separate these quantities [4, 5], but the concept is based on simplistic and arbitrary assumptions such as a frequency-independent source impedance. Further, the method unfairly lumps customers with linear and nonlinear loads and only those customers with pure resistive loads are considered not to affect voltage distortion.

The objective of this paper is propose a method to separate the contribution of the customer-side and the utility-side to individual harmonic voltages at the point of coupling. Assuming that the network configuration and load parameters are readily available, a simplified circuit composed of a Thevenin and Norton equivalents can be derived. Contributions to voltage distortion from the source-side and load-side are then determined by superposition. The method is illustrated by a numerical example which shows that the separation is highly dependent on phase angles of harmonic impedances, currents and voltages.

2. ASSUMPTIONS

In order to simplify the analysis, the following assumptions are made:

- The circuit is assumed at steady-state so that the concept of impedance in linear circuit theory can be applied. In real life, load conditions and system configuration are constantly changing. Hence, the analysis represents a snapshot in time.
- The linear current injection method is chosen due to its simplicity. In here, nonlinear loads are modeled as harmonic current sources, while linear loads, feeders, transformers and shunt capacitors are modeled by their equ-

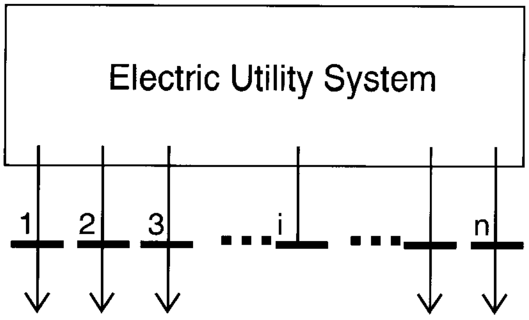


Fig. 1. Electrical Network Showing Customer Nodes

ivalent impedance at the harmonic frequency of interest. Voltage-dependent methods are avoided since they require iterative techniques and device detailed models. The linear method assumes no interaction between harmonic voltages and currents of different orders. It is possible to take into account the influence between harmonics of different orders if the so-called crossed-frequency admittance matrix [6] is known.

- Due to their importance in this subject, the harmonic current phase angles with respect to their fundamental component and the displacement angle (i.e., power factor angle at fundamental frequency) of each load aresupposed to be known. Furthermore, the harmonic power flows are negligible compared of power-frequency components so that the relative phase angles of different node voltages can be determined by conventional load flow.

3. EQUIVALENT CIRCUIT

Given a power network serving a number of customer loads as shown in Figure 1, it is desired desired to derive the equivalent circuit looking from node *i* into the network. This will provide the pre-existing condition at node *i* where a new customer will be served from. Then the objective is to quantify the harmonic voltage at the same node after this customer with known load parameters is connected to the network. Knowing the impedance of network elements and load data of all existing customers in terms of their active and reactive power consumption, the power-frequency voltage phasor at node *i* is first calculated by conventional load flow analysis. This is a necessary first step to determine the magnitude and phase angle of the Thevenin (i.e., open-circuit) voltage prior to servicing the new load. The Thevenin impedance of this voltage can be approximated from the short circuit capacity at node *i*, or more accurately determined by including all shunt elements such as capacitors and equivalent load impedances.

Both the source voltage and source impedance needs to be evaluated at each harmonic frequency of order *h*. The open-circuit voltage $V^s_{i,h}$ at node *i* is determined by the commonly known linear current injection method. In here, (a) the system infinite bus is replaced by a short circuit, (b) the network impedances including shunt capacitors are adjusted to the *h*-th harmonic frequency, (c) the linear portion of the load is represented by its equivalent shunt resistance in parallel with a equivalent shunt reactance (also adjusted at the frequency of order *h*), (d) and the nonlinear portion of the load is represented by its phasor current source. The resulting harmonic voltage is calculated by

$$V^s_{i,h} = \sum_{j=1}^{N} Z_{ij,h} I^l_{j,h} \qquad (1)$$

where $Z_{ij,h}$ and $I^l_{j,h}$ represents the transfer impedance between nodes *i* and *j* at harmonic frequency of order *h*. These impedances can be evaluated by inverting the bus admittance matrix, or by simple current division in case of radial systems.

The driving point impedance representing the Thevenin source impedance at node *i* can be derived in a similar fashion, and it is a complex function of all impedances in the network:

$$Z_{ij,h} = f(Z_{mn,h}, Z_{m0,h}) \qquad (2)$$

where $Z_{mn,h}$ and $Z_{m0,h}$ represent the series impedance of the element connecting nodes *m* - *n* and shunt impedance at node *m*, respectively. The Thevenin impedance in (2) can be calculated by injecting inter-harmonic signals at the point of common coupling and measuring the current flow into utility system and into load side [7], or by switching of network elements and detecting the changes in harmonic voltage and current [8].

The new customer load is best represented by a Norton equivalent circuit [9], where the Norton impedance $Z^l_{i,h}$ corresponds to that of the linear component of the load, and the Norton current source $I^l_{i,h}$ represents the phasor harmonic current of the nonlinear component. It is therefore important to know not only the portion of the load that is nonlinear, but also the magnitude and phase angle (relative to the fundamental component) of each harmonic current component.

When the new customer at node *i* connects to the utility supply, the circuit representing both the source-side and load-side at a harmonic frequency is shown in Figure 2. This connection will cause deviation in voltage from the open circuit voltage. The question as to how much of this deviation is caused by the source-side and how much is caused by the new load will be addressed next.

4. VOLTAGE DECOMPOSITION

Obviously the node harmonic voltage $V_{i,h}$ in Figure 2 depends on the four parameters shown, i.e., the impedance and the source of the load-side and supply-side. A logical way to decompose these signals is to apply the Superposition Principle since linearity is assumed earlier.

The voltage at the coupling point due to the supply-side acting alone is simply found by removing the source current and by voltage division,

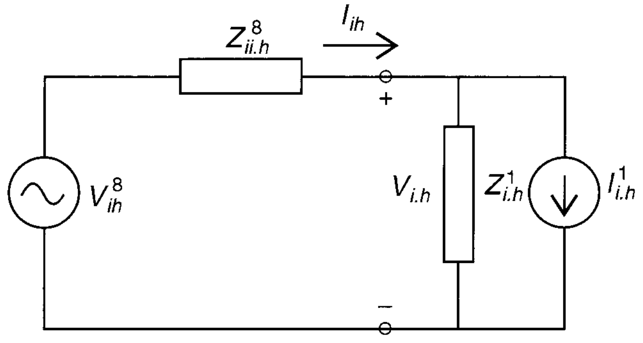


Fig. 2. Equivalent Circuit of Supply-Side and Load-Side

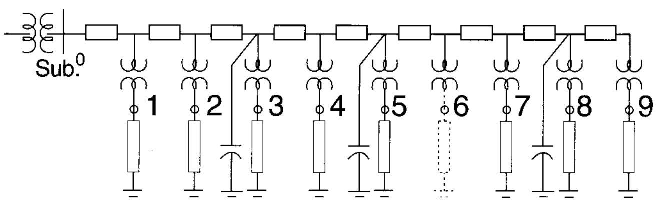


Fig. 3. Circuit Diagram of Feeder under Study

$$V_{i,h}^{supply} = V_{i,h}^{s} \frac{Z_{i,h}^{l}}{Z_{i,h}^{l} + Z_{i,h}^{s}} \tag{3}$$

Note that this above component depends on the existing harmonic level (without the new load), and the size of the linear component of load impedance relative to the source impedance. At fundamental frequency, it is clear that the source impedance is generally very small compared to the load impedance. This is not the case at harmonic frequencies, however, since parallel resonances due to shunt capacitors tend to increase the source impedance dramatically at times.

The voltage at the coupling point due to the load-side $V_{i,h}^{load}$ acting alone is similarly found by ignoring the source voltage and by current division,

$$V_{i,h}^{load} = -I_{i,h}^{l} \frac{Z_{i,h}^{s} Z_{i,h}^{l}}{Z_{i,h}^{l} + Z_{i,h}^{s}} \tag{4}$$

This component depends on the magnitude of the harmonic current generated by the nonlinear load, and the parallel combination of load and source impedances.

The resultant harmonic voltage at the coupling point is simply the phasor sum of the source-generated voltage and load-generated voltage:

$$V_{i,h} = V_{i,h}^{supply} + V_{i,h}^{load} \tag{5}$$

The magnitude of the resultant voltage depends on both the magnitude and phase angle of each of the two voltage components. Without knowledge of the relative phase angles, one can only estimate the lower and upper limit of the resultant voltage, i.e.,

$$\left| V_{i,h}^{supply} - V_{i,h}^{load} \right| \le V_{i,h} \le V_{i,h}^{supply} + V_{i,h}^{load} \tag{6}$$

These limits occur when the phasors are in phase and 180 degrees out of phase. The ratio of the two voltage components, however, is determined by knowing only the magnitudes of the source voltage, source impedance and load harmonic current,

$$\frac{V_{i,h}^{supply}}{V_{i,h}^{load}} = \frac{V_{i,h}^{s}}{Z_{i,h}^{s} I_{i,h}^{i}} \tag{7}$$

From the point of view of individual harmonic content, the effective value of the load-side contribution can be defined as the difference between the magnitude of the resultant voltage and the source-side contribution, i.e.,

$$\left(V_{i,h}^{load} \right)_{eff.} = V_{i,h} - V_{i,h}^{supply} \tag{8}$$

Note that the above effective value can be negative, a case where the load-side acts as a compensator. With regard to the total harmonic distortion, it is suggested that the load-side contribution to this parameter be the difference between the THD of the resultant voltage and that of the voltage when the supply-side source acts alone:

$$\text{THD}_{load} = \text{THD}_{total} - \text{THD}_{supply} \tag{9}$$

The following example illustrates the proposed decompositions.

5. NUMERICAL EXAMPLE

Consider a radial distribution feeder [10] with eight connected customers, and a ninth customer is about to be connected at node 6 as shown in Figure 3 below. The network and load data are as follows: Substation transformer rating: 10 MVA 138/25 kV with 5.5% impedance (R = 1% and X = 5.4%); single-phase distribution transformer rating: 500 kVA 14.4 kV/240 V/120 V with 3% impedance (R = 1% and X = 2.8%); impedance of feeder sections is given in Table 1 below; the capacitor sizes are 600 KVAr at each of the three busses; the load data in terms of real and reactive power requirement (P_j, Q_j) also given in Table 1.

In terms of nonlinearity, it is a assumed that 30% of the load at nodes 5 and 7 is nonlinear. Further, the portion of these nonlinear loads is assumed to generate harmonics of the square-wave type waveform up to the 19-th order. i.e.,

$$I_{j,h} = I_{j,1} / h, \quad h = 3,5,\ldots,19 \quad j = 5,7 \tag{10}$$

Table 1. Load and Feeder Data

Sec. i-j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (KVAr)
0 - 1	0.123	0.413	1350	450
1 - 2	0.014	0.605	960	330
2 - 3	0.746	1.205	1200	750
3 - 4	0.698	0.608	1050	360
4 - 5	1.983	1.627	1290	660
5 - 6	0.905	0.788	—	—
6 - 7	2.055	2.316	990	210
7 - 8	1.795	2.716	750	480
8 - 9	2.343	2.026	540	300

Note that these harmonic currents are in phase with the fundamental component. Hence, their phase angle relative the common reference is $h(\theta_{Vj} + \theta_{Ij})$.

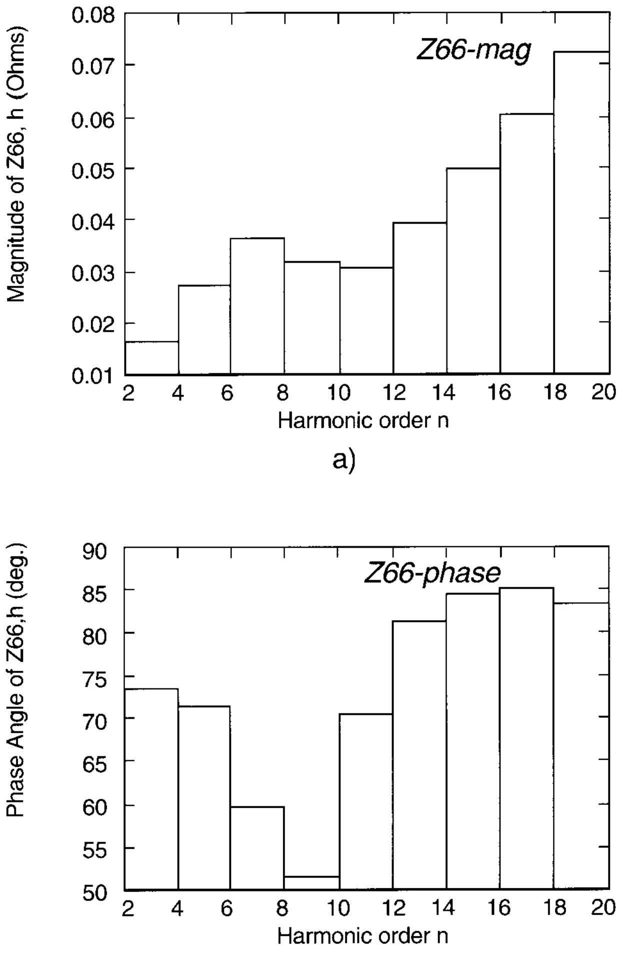
With the above data, one can determine the Thevenin equivalent of the supply-side at node 6. Starting with a voltage of $1.02\angle 0^{\circ}$ pu at the substation terminals (i.e., node 0), a load flow study resulted in a voltage of $V_{6,1} = .98\angle{-2.3^{\circ}}$ pu (based on 240 V) at node 6, and the Thevenin impedance at fundamental frequency is found to equal $Z_{66,1} = (2.8 + j\ 5.4)10^{-3}\ \Omega$. The magnitude and phase angle of the Thevenin voltage source $V^{s}_{6,h}$ and Thevenin impedance $Z^{s}_{6,h}$ are also calculated accordingly and their values are shown in Figures 4 and 5. These figures give a complete characterization of the source-side circuit at harmonic frequencies.

The resultant voltage total harmonic distortion (THD) at node 6under open-circuit condition is 1.77%, a value that is considered acceptable by any standard. Now the new customer installs a 750 kVA load at .85 power factor lag at node 6. It is assumed that 20 % of this load is nonlinear, and the harmonic currents generated are of the same types as those of loads at nodes 5 and 7, i.e., as described in equation (10). At power frequency, this new load results in a power-frequency voltage drop from $.98\angle{-2.3^{\circ}}$ to $0.96\angle{-3.0^{\circ}}$ pu at node 6.

Node 6 voltage is also recalculated at each characteristic harmonic and the new values are shown in Figure 6 where the harmonic voltages components caused by the source-side and load-side are also shown. Note that both components are about equal at low-order harmonics, but the load-side dominates at harmonic order above the 9th. Partial cancellation between the two components can also be noticed at harmonics of order the 13, 15 and 17. The total voltage THD went up to 3.75%, and the supply-side's contribution is $THD_{supply} = 1.67\%$. Hence, it can be said that the load-side contributed a $THD_{load} = 2.08\%$, or 55% or the total value.

The amount of harmonic cancellation among the load- and supply-side is highly dependent on the harmonic current phase angle and customer load power factor angle. To illustrate this point, the new customer power factor is altered to 0.8 while the rest of the load characteristics are kept the same. The new harmonic components are shown in Figure 7. With this lower power factor, the total voltage THD dropped to 3.33%, half of which is contributed by the load-side. This is due to more significant cancellations that occur at most of the harmonic frequencies.

A further analysis considers the effect of power factor correction capacitor placement by the customer on the supply- and load-side harmonic voltages. When the new customer installs a 75KVAr/phase shunt capacitor at the load terminals to improve the load power factor from 85% to 98% lag, the total voltage THD increased dramatically to 5.41%, as seen by the individual harmonic magnitudes in Figure 8. Of this distortion, 2.04% is caused by the supply-side and a large value of 3.37% is caused by the load-side. This capacitor installation caused an increase in every harmonic component of both the load-side and supply-side. The most increase occurs at the 15-th harmonic that more than tripled.



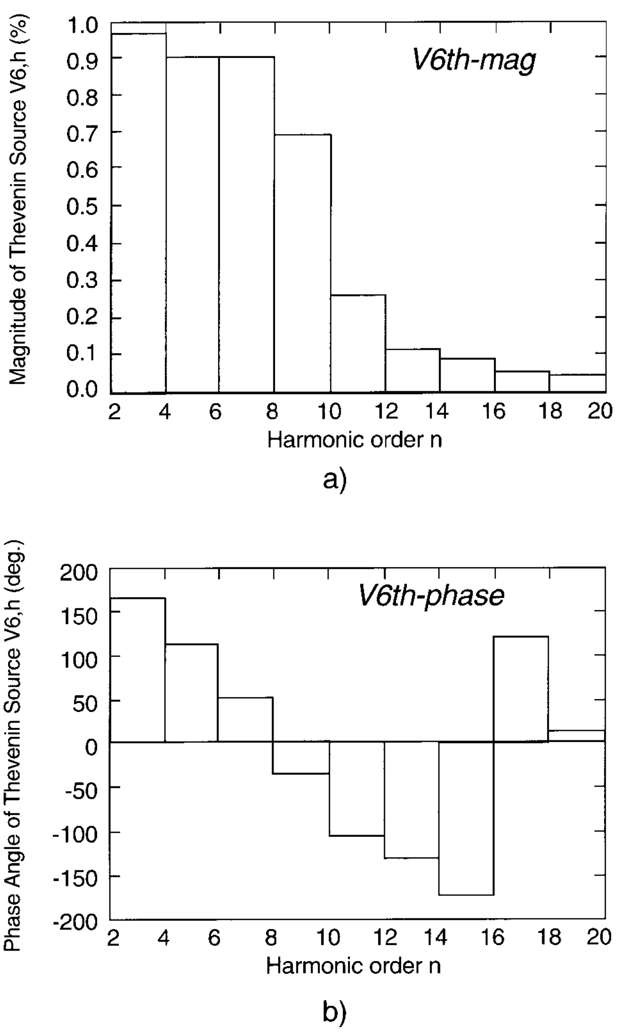
a)

b)

Fig. 4. Variation of (a) Magnitude and (b) Phase Angle of Equivalent Source Impedance a Node 6 with Harmonic Order n

5. CONCLUSION

This paper presented a simple method to decompose the voltage distortion into supply-side and load-side components at a tie point with an individual customer operating nonlinear loads. The method is based on the superposition principle, and assumes a steady-state condition where the parameters of the Thevenin equivalent of the source-side and the Norton equivalent of the load-side are known or can be calculated at each harmonic frequency. It is emphasized that relati-



a)

b)

Fig. 5. Variation of (a) Magnitude and (b) Phase Angle of Equivalent Voltage Source at Node 6 with Harmonic Order n

ve phase angles of source harmonic voltages and load harmonic currents, and shunt capacitor locations have a dramatic effect on the resulting harmonic magnitudes. It is hoped that the method will shed some light on the complexity involved in determining individual responsibilities to voltage distortion in electrical distribution systems.

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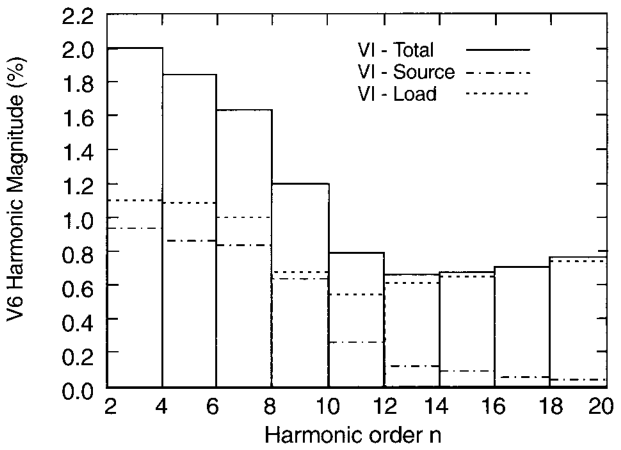

Fig. 6. Resultant Voltage Harmonic Levels at Node 6 with New Customer (85% Power Factor)

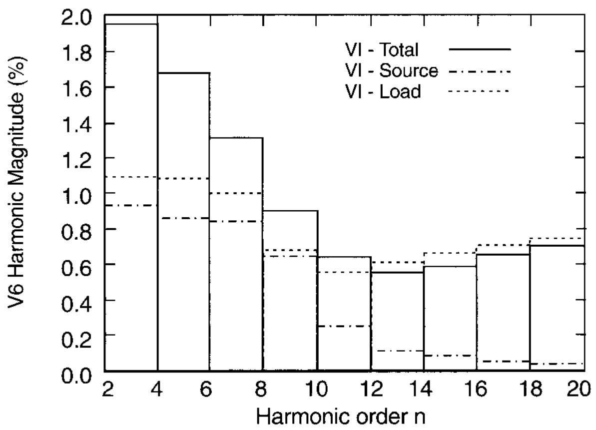

Fig. 7. Resultant Voltage Harmonic Levels at Node 6 with New Customer (80% Power Factor)

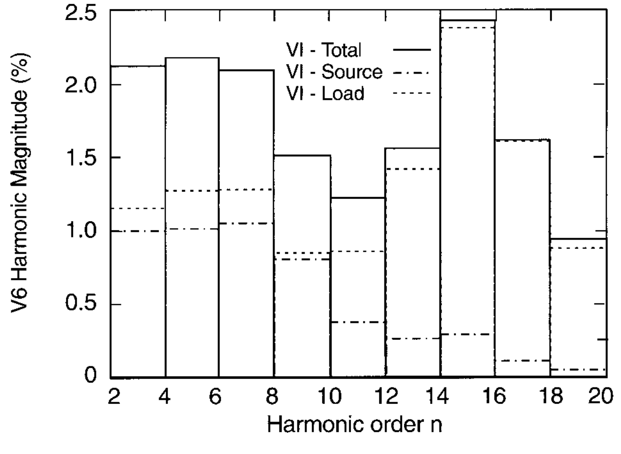

Fig. 8. Resultant Voltage Harmonic Levels at Node 6 with New Customer and PF Correction Capacitor

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