Corona Modeling for the Transient Analysis, Steady State and Corona Loss Performance of Transmission Lines

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Summary: This paper addresses the steady state analysis and the electromagnetic transients of overhead power transmission lines exhibiting corona. After a brief survey of the currently available modeling techniques, an alternative approach to the simulation of the corona discharge is presented. It is based on adding voltage-dependent current sources at prespecified nodes within the line’s lumped, ladder-shaped, equivalent circuit including an adequate number of sections. The characteristics of these current sources can be determined using the usually available corona power loss measurements under power frequency sinusoidal voltage conditions. The line model is used for finding its transient response using time domain techniques. Several voltage excitation waveforms and magnitudes as well as different line’s loading conditions are investigated. As a further application of the suggested approach, the minimization of the total corona losses of a long shunt compensated line will be considered as an additional criterion in the process of identifying the appropriate rating and location of the compensating coil primarily used for improving the line’s voltage profile.

1. INTRODUCTION

The electromagnetic transients in power networks in general, and in transmission lines in particular, have been the topic of many interesting investigations such as [1-16]. The applied techniques include, among others, the time-domain [1-4], the Laplace s-domain and the traveling wave analyses [1,2]. The time-domain approach can take into consideration important nonlinear phenomena such as the corona discharge and core saturation [6-16]. The linearity, on the other hand, is a prerequisite to the use of the s-domain methods. The advantage of the traveling wave analysis and the related lattice diagrams is specially recognizable in situations involving lossless lines with ohmic terminations. The recent research topics in the area of electromagnetic transients include the full consideration of the circuit parameters’ frequency dependence, the presence of earth return and the ground wires, and the consideration of eventually existing nonlinearities, just to name a few [3,6-16]. This paper is a contribution to the transient analysis of overhead transmission lines under corona. References such as [11, 14] indicate that neglecting corona would introduce a considerable error in the magnitudes and the time waveforms of the transient stresses. The phenomenon of corona on power lines is linked to the partial electric breakdown of the air surrounding the line conductors. This breakdown occurs if the electric field on the conductor’s surface, at standard weather conditions, reaches the theoretical value of 30kV/cm (peak value). The corresponding corona current is, therefore, equal to zero unless the instantaneous conductor voltage exceeds a certain onset value, above which the corona current increases rapidly with the voltage. The literature includes several techniques for simulating the line corona. A basic approach suggests the modeling of corona using combinations of equivalent batteries, diodes and linear resistors, together with several associated logical conditions, to be added to the corona regions along the line[4]. Their values can be determined from the experimentally obtained dependence of line’s corona losses on the line’s voltage under power frequency sinusoidal conditions [5]. Another approach is to consider the conductor capacitance increases from the initial “geometrical” value to larger values under corona [2,8]. This implies the consideration of the line shunt capacitance (or equivalently its effective radius) as one of the lines state variables in the transient analysis [4]. In an interesting investigation, reference [12] addresses an approach based on modifying the line’s telegrapher equations and their solution using finite differences in characteristic co-ordinates.

Most of the above mentioned analyses are based on the numerical solution of a large number of algebraic and differential equations in the state variables of the line’s equivalent circuit, i.e. the node voltages and inductor currents. Accordingly, the number of the state variables is double the number of the assumed line sections. Any effort towards increasing the solution efficiency will be manifested in an improved accuracy and/or a reduction in the required computational resources. This paper suggests an efficient straight forward representation of the line sections’ corona discharge using circuit elements of strongly nonlinear voltage-current characteristics. The suggested approach eliminates the need for evaluating any logical expressions during the solution of the system’s set of equations. The paper includes four parts:

— Description of the proposed corona model.
— The model validation through comparison with the results available in the literature.
— The application of the suggested representation in solving several case studies involving line transients.
— Adopting the presented model for investigating the impact of the long line compensation involving shunt-connected coils, used primarily for controlling the line’s voltage profile, on the line’s overall corona losses. Results will imply the possible minimization
of these losses as an additional objective of the shunt compensation.

2. METHOD OF ANALYSIS & SAMPLE RESULTS

2.1. The Corona Model

The suitability of using the assumption

\[ i_{\text{corona}} = a v_{\text{corona}}^k \]  

for describing the voltage-current characteristic of corona discharge can be clearly recognized from the corresponding plot in Fig.1. In Eq.(1), \( i_{\text{corona}} \) and \( v_{\text{corona}} \) denote the corona current per meter, and the voltage across the considered transmission line section, respectively. The value of \( k \) should be a positive integer number and greater than 1. The constant \( a \) depends on the atmospheric conditions, the line design and the conductors' surface conditions. It can be seen that the current \( i_{\text{corona}} \) is almost zero for voltage magnitudes of \( v_{\text{corona}} \) less than about 160kV. In other words, the above equation can model a line section of a corona onset voltage of 160kV. For a 1m line section, and an applied sinusoidal voltage \( v(t) = V_{\text{max}} \sin(2\pi ft) \), the current waveform is

\[ i(t)_{\text{corona}} = a V_{\text{max}}^k \sin(2\pi ft)^k \]

where \( t \) and \( f \) are the time in seconds and the power frequency in Hz, respectively. Using Mathematica, the average corona power loss \( P_{\text{corona}} \) is therefore

\[ P_{\text{corona}} = \frac{1}{f} \int_0^f a V_{\text{max}}^k \sin(2\pi ft)^k dt = \frac{2a V_{\text{max}}^k (1 + K/2)}{2\Gamma(1.5 + K/2)\sqrt{\pi}} \]

where \( \Gamma(x) \) denotes the Gamma function of the argument \( x \).

The two constants \( a \) and \( K \) can be easily determined from the corona loss measurements at two different voltages. Fig. 2, for instance, shows the power loss in Watts per meter for a 3-phase line at the two rms line voltages 480 and 400kV, versus the exponent \( K \), for \( a = 0.5 \times 10^{-31} \). It indicates that the value \( K=5 \) is suitable for the two corresponding 3-phase power loss measurements of 57 and 170W/m, respectively.

2.2. The Sinusoidal Steady State Corona Performance of a Short Line Section

The plots in Fig. 3 describe the steady state corona performance of a single-phase line section having the surge impedance 400Ω, conductor type 1.4 inch, ACSR. The capacitance per m is 8.55pF. The 50-Hz sinusoidal voltage has a peak value of 400kV and starts with a zero crossing at \( t=0 \). The corona current has a much distorted waveform and appears only in the time regions during which the instantaneous voltage exceeds the onset value 160kV, plot (a). The total current including the corona and the purely sinusoidal (with a phase lead of \( \pi/2 \)) capacitive components is shown in plot (b). The curve (c) depicts the instantaneous corona losses in Watt/m. Its average value

is approximately 57Watt/m, as expected. The relationship between the applied voltage and the electric charge (which is not exactly sinusoidal) is given by the plot (d). It should be noted that without corona, both curves should be in phase. The clearly recognizable small phase shift of several degrees is due to the presence of corona.In the absence of corona, a parametric plot of the voltage and (the purely capacitive) current should be a smooth ellipse. The clear deviation from this ideal situation can be observed in plot (e), with corona. Without corona, the charge is proportional to the voltage. Accordingly, the voltage-charge parametric relation should be a straight line passing through the origin. Due to corona, the plot (f) indicates a completely nonlinear voltage-charge relationship. It depicts also a hysteresis effect. The area of the hysteresis loop is proportional to the corona loss.

2.3. The Transient Response of a Short Line Section to a Double-Exponential Voltage Input, Under Corona

The effect of corona on the line’s response to the double-exponential voltage of the waveform

\[ v(t) = 600 \left( e^{-\frac{t}{0.4}} - e^{-\frac{t}{0.5}} \right) \]

\[ (3) \]
is depicted in the four plots of Fig. 4. The peak voltage is approximately 600kV, way above the conductor’s onset corona voltage of 160kV. The plots (a) and (b) indicate peak corona current and power loss/meter of about 3.2mA and 1.85kW, respectively. From plot (c), the charge reaches a maximum value of 4.8µColumb. Again here, a slight hysteresis effect can be observed in the voltage-charge parametric plot illustrated in plot (d).

2.4. The Transient Response of a Transmission Line to a Step Source Voltage, Under Corona

The plots in Fig. 5 describe the electromagnetic performance of a 40-km line under corona. The line is simulated in time domain using 20 sections, 2km each. It has the following electrical parameters: resistance 0.045 Ω/km, inductance 1.346µH/m and capacitance 8.55pF/m. Its shunt conductance is neglected. The line is energized at its sending end by a 400kV step voltage source of negligibly small internal
The corona is simulated according the previously described model as a voltage dependent shunt current for each of the 20 sections. The problem variables are 20 voltages (at points 2km, 4km, 6km, etc. far from the sending end) and the series currents through the line conductor at the same locations. A total of 40 differential equations are solved numerically using the software Mathematica. The results are given for three line receiving end loading conditions. The plots (a), (b) and (c) depict the voltages, the currents as well as the corona current (per section) at the sending end, the line middle point and at the line receiving end, respectively, when the line is open-circuited at the receiving-end, Case (A). It should be noted that the given current at the line end is the one flowing through the last 2 km line section, which is identical with the corresponding corona current under no load. The initial value of the sending end current in the three cases is 1000 A, since the line has 400Ω surge impedance. The corona current of the first 2km near the source is seen to be about 1.1A. The corresponding values for the sections at the line middle point and the receiving end are 18 and 25A/section, respectively. The voltage plot pertinent to Case (B), i.e. curve (d) of Fig.5, indicate constant values of 400kV and zero at the sending and receiving ends, respectively, as expected. The voltage at the line’s middle point oscillates with diminishing amplitude around the final value of 200kV. The interesting plot (e) of the three currents shows the general trend of an almost linear increase at an average rate of about 7.5A/µs. This agrees with the expected value 

\[ \frac{di}{dt} \approx \frac{400000}{L_{line, total}} \]

Its final value is expected to be about 

\[ \frac{400000V}{R_{line, total}} \approx \frac{400000V}{1.8} \approx 222.22kA \]

The corona currents per section at the sending and receiving ends are 1.1A and zero, respectively. At the line’s middle section, the corona currents starts with approximately 1 A/section and decreases gradually with time. The plots (g), (h) and (i) in Fig.5 pertinent to the Case (C) describe the line’s transient performance with proper termination, i.e. 

\[ Z_{load} = Z_o = 400Ω \]

Due to the absence of any wave reflections, all voltages, longitudinal and corona currents at all line sections are equal: 400kV, 1kA and about 1A, respectively. The time delays, magnitudes and wave

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**Fig. 4.** The corona performance of a 1m long section of a single phase line subjected to the double-exponential voltage:

\[ v(t) = 600(e^{-0.405t} - e^{-0.0405t}) \text{kV} \]

where \( t \) is the time in µsec.
Fig. 5. The transient response of the 40km line to the application of a 400kV step voltage for three different receiving-end terminations:

(A) Unloaded Line           (B) Short-Circuited Line            (C) Properly-Terminated Line

a) Voltages
b) Currents
c) Corona Currents
Case (A); Open-Circuited Line
d) Voltages
e) Currents
f) Corona Currents
Case (B); Short-Circuited Line
g) Voltages
h) Currents
shapes of the voltages and currents at the three locations are in agreement with the results obtainable from the wave techniques using the lattice diagrams.

2.5. The Transient Response of a Transmission Line to a Double-Exponential Voltage Source, Under Corona

The line’s transients following the application of a double-exponential voltage source \( v(t) = 900 \left( e^{-\frac{t}{0.05}} - e^{-\frac{t}{0.005}} \right) \) kV at the sending end is illustrated in Fig. 6. Plots (a), (b), (c) and (d) deal with the case of proper termination \( (Z_{\text{Load}} = Z_0) \).

(i) Corona Currents
Case (C); Line Terminated by \( Z_{\text{Load}} = Z_0 \)

Fig. 5. The transient response of the 40km line to the application of a 400kV step voltage for three different receiving-end terminations: (A) Unloaded Line; (B) Short-Circuited Line; (C) Properly-Terminated Line

Fig. 6. The transient response of the 40km line to the application of a double-exponential voltage source \( v(t) = 900 \left( e^{-\frac{t}{0.05}} - e^{-\frac{t}{0.005}} \right) \) kV at the sending end is illustrated in Fig. 6. Plots (a), (b), (c) and (d) deal with the case of proper termination \( (Z_{\text{Load}} = Z_0) \).

(a) Voltages \( (Z_{\text{Load}} = Z_0) \)

(b) Currents \( (Z_{\text{Load}} = Z_0) \)

(c) Corona Currents \( (Z_{\text{Load}} = Z_0) \)

Plots are given for locations 0, 10, 20, 30 and 40km far from source

(d) More details of the voltage plot (a) after discarding the line delays \( (Z_{\text{Load}} = Z_0) \)
Plots are given for locations 0, 10, 20, 30 and 40km far from source

(e) Receiving-end voltage for \( Z_{\text{Load}} = 2Z_0 = 800 \Omega \)
To be compared with Fig.4(d) of Reference [4]
Fig. 7. The equivalent circuit of a shunt-compensated long line under Corona.

2.6. The Steady State Analysis of a Long Shunt-Compensated Line, under Corona

2.6.1. The Voltage and Current Distributions

The two cascade-connected two-port networks in Fig. 2.6 represent the two sections of a long transmission line of total length \( D \), corresponding to a total phase delay of \( \theta \). At 50-Hz, the angle \( \theta \) is approximately 6 degrees/100km. The line’s series ohmic losses and shunt conductance are assumed negligibly small, so that its surge impedance is \( Z_0 \) is pure real. A shunt compensating coil is connected at a distance \( y \) measured from the source. The parameters \( A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2 \) are the transmission parameters of the two sections [17]. They are defined by:

\[
\begin{align*}
A_1 &= D_1 = \cos[y\theta / D] \\
B_1 &= jZ_o \sin[y\theta / D] \\
C_1 &= -jZ_o \sin[(D-y)\theta / D] \\
D_1 &= \cos[(D-y)\theta / D] \\
B_2 &= jZ_o \sin[(D-y)\theta / D] \\
C_2 &= \cos[(D-y)\theta / D] \\
D_2 &= \cos[(D-y)\theta / D]
\end{align*}
\]

Applying the usual two-port equations for each of the two line sections, together with

\[
I_{r2} = -(j/A_1) V_r \quad \text{and} \quad I_{c0} = -(j/k) V_y.
\]

Fig. 8. The per unit voltage distribution (top, based on the source voltage \( E \)) and the per unit current distribution (bottom, based on \( E/Z_0 \)) along an unloaded 500km 400kV line for five different coil ratings: \( k = 0.25, 0.50, 0.75, 1.00 \) and 1.25 per unit (the lowest curve), installed at the line’s midpoint. It should be noted that without compensation (i.e. \( k = 0 \)), the receiving end voltage will be \( 1 / \cos(\pi/6) = 1.223 \) per unit based on the source voltage \( E \). In other words, the voltage along the line in this case will range between 1.00 and 1.223 per unit. The

for the conditions at the receiving end and the shunt compensating coil, respectively, the voltage and current at any location \( x \) along the left side line section can be easily found. The equations should clearly distinguish between locations to the left or the right of the coil location. The voltage and current distributions depend, among others, on the load at the receiving end (expressed by the parameter \( m \) in per unit), the coil location (expressed by the variable \( y \) in meters or occasionally in per unit of the total line length \( D \)), and its size (in terms of the parameter \( k \) in per unit). The value \( m=0 \) corresponds to the no-load condition, while \( m=1 \) describes the case of inductive line loading of \( V_r^2 / Z_o \) MVAR per phase. The value \( m=+j1 \) represents the case of purely resistive surge impedance loading. The parameter \( k \) can assume the special values \( k=0 \) and \( k=1 \) for the cases of no compensating coil, and the presence of a coil of rating \( V_r^2 / Z_o \) MVAR per phase, respectively. Each of the two plots in Fig. 8 depicts the voltage or current distributions along the line for five different ratings of the shunt compensating coil: \( k=0.25 \) (the highest curve), 0.50, 0.75, 1.00 and 1.25 per unit (the lowest curve), installed at the line’s midpoint. It is noteworthy that without compensation (i.e. \( k=0 \)), the receiving end voltage will be \( 1 / \cos(\pi/6) = 1.223 \) per unit based on the source voltage \( E \). In other words, the voltage along the line in this case will range between 1.00 and 1.223 per unit. The
corresponding sending end (source) current can be calculated as \( \tan \left( \frac{\pi}{6} \right) = 0.5095 \) per unit (capacitive), based on \( \frac{E}{Z_0} \).

As expected, all of the five current distributions start with value zero at the receiving end. The discontinuities at \( y = 0.5 \) per unit are due to the inductive current of the compensating coil. The source current changes from 0.5095 per unit (capacitive) in the uncompensated case to about 0.25 per unit (capacitive) at a shunt compensation level \( k = 0.25 \) and becomes approximately 0.55 per unit (inductive) for \( k = 1.25 \).

**2.6.2. The Line’s Corona Loss Performance & Minimization**

As an example, Fig. 9 depicts a three-dimensional plot and a corresponding contour plot relating the total corona loss of the unloaded line \( m = 0 \) as a function of both the rating \( k \) and location \( y \) of the compensating coil. Other loading conditions can be easily investigated by substituting the relevant value of \( m \) in equations (4) and (5).

Without compensation, the total corona losses of the unloaded 500-km line is about 17MW. This corresponds to an average power loss along the line of 17MW/500km = 34Watts/m. From the 3-D plot, the total losses decreases to a minimum of about 5MW with shunt compensation. The contour plot indicates that there are special values of the coil rating \( k \) and location \( y \) that can lead to this minimum. More practically applicable values can be identified, if certain constraints on the voltage profile along the line are properly incorporated in the minimization procedure. The plots in Fig.10 show the effect of the location \( y \) of the a shunt compensating coil having a per unit compensation level \( k = 0.8 \) on the line’s total 3-phase corona losses. The curve (a) indicates that these losses decrease from about 17MW for the theoretically insignificant case \( y = 0 \), to a minimum of about 5.9MW if the coil is connected at \( y = 0.90 \) per unit (i.e. at 50km far from the receiving end). The resulting voltage and current distributions along the line, together with a comparison with the corresponding profiles in the uncompensated case, are illustrated in plots (b) and (c), respectively.

Plot (c) shows the discontinuity in the current distribution at \( y = 0.90 \) per unit due to the coil current. Plot (d) shows the distribution of the corona power loss per meter along the unloaded line without compensation (upper trace) and with a compensating coil having \( k = 0.80 \) at \( y = 0.90 \) per unit (lower trace). Both curves indicate the same corona loss (about 20 Watt per meter) at the sending end, since the voltage is equal to the source value \( E \) in both cases. The voltage rise of the uncompensated line leads to a corresponding increase in the corona loss to reach approximately 44Watt/m at the receiving end. The improvement in the voltage profile due to the shunt coil results in the smaller power loss per meter of about 6 watt at the receiving end, as indicated by the lower curve of plot (d).

**3. CONCLUSIONS**

1. The paper addresses the steady state performance and transient performance of power lines exhibiting corona. It presents an approach to the simulation of the corona
1. The suitability of using the assumption \( i_{\text{corona}} = a \cdot v_{\text{corona}} \) for describing the voltage-current characteristic of corona discharge was demonstrated, where \( i_{\text{corona}} \) and \( v_{\text{corona}} \) denote the corona current per meter, and the voltage across the considered transmission line section, respectively. For a particular 400kV line, the appropriate values \( a = 0.5 \times 10^{-32} \) and \( K = 5 \) were determined.

2. The steady state corona performance of a short single-phase line section shows that the applied sinusoidal voltage and the electric charge are not exactly in phase. Without corona, a parametric plot of the voltage and (the purely capacitive) current is expected to be a smooth ellipse. Under corona, a deviation from this ideal situation could be observed.

3. The voltage-charge parametric relation in the absence of corona is a straight line passing through the origin. The corresponding plot under corona indicates a strongly nonlinear relationship with a hysteresis effect. The area of the hysteresis loop is proportional to the corona loss. Without corona, a parametric plot of the voltage and (the purely capacitive) current is expected to be a smooth ellipse. Under corona, a deviation from this ideal situation could be observed.

4. The voltage-charge parametric relation in the absence of corona is a straight line passing through the origin. The corresponding plot under corona indicates a strongly nonlinear relationship with a hysteresis effect. The area of the hysteresis loop is proportional to the corona loss. Without corona, a parametric plot of the voltage and (the purely capacitive) current is expected to be a smooth ellipse. Under corona, a deviation from this ideal situation could be observed.

5. The effect of corona on the line’s response to a double-exponential voltage of a peak value of about 600kV was investigated. The peak corona current and power are about 3.2mA and 1.85kW per meter, respectively. The charge reaches a maximum value of 4.8µColumb per meter. A slight hysteresis effect can be observed in the voltage-charge parametric plot.

6. Results illustrating the electromagnetic performance of a 40-km line under corona are presented. The line is simulated in the time domain using 20 sections, 2km each. It is energized by a 400kV step voltage source. Various line loading conditions are considered.

7. The transients due to a double-exponential voltage source of a peak value about 900kV are also investigated for the case of line termination by its surge impedance. The peak value of the voltage wave decreases to about 500kV upon arrival at the receiving end. The corresponding currents decrease from approximately 2kA at the source, to about
1kA at the load. The peak corona current/section at the source, the line’s midpoint and at the last section closest to the load is approximately 50, 8 and 5A, respectively.

8. In order to validate the suggested model, the receiving end voltage is determined for the 900 double-exponential voltage source, with the line terminated by double its surge impedance. The results agree quite well with the corresponding solution available in the literature based on a classical modelling of corona.

9. As a further application of the suggested approach, the possible reduction in the total corona losses of a long shunt-compensated line is considered and recommended as an additional criterion in the process of identifying the appropriate size and location of the compensating coil.

10. Plots for the line’s total corona loss as a function of both the coil’s rating and location are presented. Without compensation, the total corona losses of the unloaded 500-km line is about 17MW. They can be reduced to about 5 MW with proper shunt compensation. There are optimal values for the coil rating and location that can lead to this minimum.

REFERENCES