Transfiguration of Supplied-by-HV/MV Transformer Network to the Supply Radius

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Summary: A HV/MV transformer supplies a few power lines of known parameters and loads. These power lines can be substituted with the supply radius, at the end of which there is the equivalent load-receiver representing the entire load being transmitted by the transformer. The R and X parameters of this substitute supply radius are needed to calculate adjusting parameters of the voltage regulator which controls the HV/MV transformer. The presented method enables simple determining of the supply radius parameters *R* and *X*.

Key words:
power network transfiguration, voltage regulation, voltage drop

1. INTRODUCTION

The voltage value is the principal quality index of electrical energy being supplied to its consumers. An unsuitable voltage makes an energy consumer suffer loss; voltage decreasing below its rated (optimal) value causes (among others):
— decreasing of the light stream, being emitted by sources of the light,
— decreasing of the devices efficiency,
— overheating of motors.

On the other hand, voltage increasing above its rated value causes:
— shortening of the devices lifetime,
— worsening of the insulation state.

According to the standing (till the end of the year 2003) regulations, the electrical energy consumers should be supplied with the voltage containing in the range [3]:
— for country-terrain power network:
$0.90\ U_N \leq U_0 \leq 1.05\ U_N$,
— for urban power network:
$0.95\ U_N \leq U_0 \leq 1.05\ U_N$.

The Minister of Economy Order of 14.09.1999, concerning the obligatory application of suitable Polish Rules, has brought into practice the Polish Rule PN-IEC 60038 "Standardized voltages IEC". The main decisions of this Rule are as follows:
— raising of the rated voltage from 220/380 V up to 230/400 V in the low voltage power network to be finally realized till the end of the year 2003,
— the range of admissible voltage is to be equal to 230 V ± 10% i.e. from 207 V to 253 V since the beginning of 2003.

Keeping the voltage in the determined admissible range ensures the high quality electrical energy for its consumers, allows to achieve the high reliability of the electric power system functioning, and minimizes the transmission and distribution energy loss [2].

2. REGULATION OF MV POWER NETWORK SUPPLYING VOLTAGE

A HV/MV transformer (being named below supply point – SP) supplies several MV power lines, which carry electrical power over to the specified number of ***load-receivers***. The term ***load-receiver*** means an electric device which consumes the electrical power; the MV/LV transformers are the main load-receivers in the MV power line (the power capacitors and local (wind or water powered) generators being included). To simplify the question of the voltage U_Z regulation in the SP, the analyzed power network has been projected as the simple circuit, presenting a supply radius, which supplies the equivalent load-receiver representing the HV/MV transformer load (Fig. 1).

The notations used in Figure 1 have been explained below:

U_{T0} — program of the planned – for the regulator – voltage value,
U_Z, U_0 — supplying and consuming voltage, respectively,
R_Z, X_Z — resistance and reactance of the supply radius,
$S_Z = P_Z + j\,Q_Z$ — apparent power being consumed by the equivalent load-receiver.

The load, represented by the equivalent load-receiver connected to the point A, is equal to the apparent power being carried over by the SP into the considered real power network. It has been assumed that the voltage drop arising in the supply radius (of parameters: R_Z, X_Z), providing this equivalent load-receiver with the electrical power, corresponds with the average voltage drop in the power network.

Assuming that the real power network parameters: R_L, X_L — representing the impedance "being seen" from the SP – are known, the following quotient is defined on their basis:

$$m = \frac{X_L}{R_L} \cong \frac{X_Z}{R_Z} \tag{1}$$

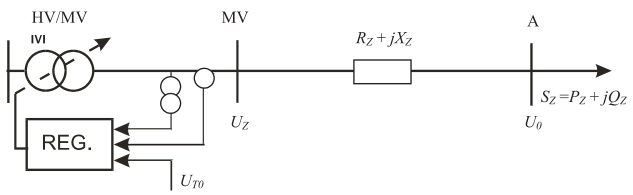


Fig. 1. The power network model – regulation of voltage

3. THE REAL POWER NETWORK IMPEDANCE [1]

The operations presented below intend to determine quotient *m* (1) for the real power net-work being supplied by the considered SP. In order to calculate this network impedance ("being seen" from the SP medium voltage side), the following actions should be applied:

— determining the main path for each *gth* power line ($g = 1, 2, ..., G$, G – number of lines),
— replacing of G main paths with the radius (arc) supplying the load-receiver, which represents the load of all the lines; resistance and reactance of the substitute supply radius are equal to R_Z ($\approx R_L$) and X_Z ($\approx X_L$), respectively.

A path in the power line is defined as a series of arcs joining the supplying node (type S) with the finishing node (type F) or with the dissecting (dividing) node (type D). The last two types of nodes belong to the category named generally ***the end nodes***. In the dissecting node there is a break which separates the two paths from each other (thus, the MV power lines are one-sidedly supplied). The main path is the one in which the voltage drop achieves the maximal value.

Each path of *gth* power line (index *g* has been neglected below) consists of a number of arcs: its first arc starts from the supplying node, and the last one finishes in the end node. The succeeding relation:

$$N_{s,v} = N_{f,v-1}, \qquad v = 2, 3,..., V_w$$

where:
$N_{s,v}$ — start node identifier (name or number) of *vth* arc,
$N_{f,v}$ — end node identifier (name or number) of *vth* arc,
v — index of arc in a path ($v = 1, 2,..., V_w$),
V_w — number of arcs in *wth* path,
w — index of path in a power line ($w = 1, 2,..., W$),
W — number of paths in *gth* power line ($g = 1, 2,..., G$),
means that the start node of *vth* arc is ***the same*** as the end node of $(v–1)th$ arc.
The resistance R_w of *wth* path is equal to:

$$R_w = \sum_{v=1}^{V(w)} \frac{l_v}{\gamma s_v} \tag{2}$$

and its reactance – to:

$$X_w = 10^{-3} \sum_{v=1}^{V(w)} x_{0v} l_v \tag{3}$$

where:
l_v — length of *vth* arc, m,
γ — conductivity, m/Ω mm²,
s_v — section of *vth* arc, mm²,
x_{0v} — specific reactance of *vth* arc, Ω/km.

The total voltage drop depends (among others) on the parameters: R_w, X_w. A greater load corresponds prevalently with the longer path, and a smaller one — with the shorter path; this implies a procedure, searching for the main path, may be limited to the algorithm:

$$M_{max} = \max_w \left\{ \sum_{v=1}^{V(w)} l_v \left(\frac{1}{\gamma s_v} + 10^{-3} x_{0v} \right) \right\} \tag{4}$$

Each power line should be a ***homogeneous*** one i.e. it should consist either of cable arcs or overhead arcs. In this case the formula (4) becomes simpler:

$$M_{max} = \max_w \left\{ \sum_{v=1}^{V(w)} \frac{l_v}{\gamma s_v} \right\} \tag{5}$$

According to (5), the path of the maximal measure M_{max} is the main path (its number of arcs is equal to V_{max}). The average conductor section σ in the main path is equal to:

$$\sigma = \sum_{v=1}^{V_{max}} l_v s_v \cdot \left(\sum_{v=1}^{V_{max}} l_v \right)^{-1} \tag{6}$$

Next, each branch of the main path should be replaced by the ***reducing load-receiver***, connected to the suitable branching node. This ***reducing*** load-receiver represents the load being the sum of the power being consumed by the load-receiver connected to the branching node, and of the power flowing through this node into the branch (summation concerns both the active and reactive power, separately). Consequently, the main path (of *gth* power line) looks like the power circuit presented in Figure 2; P_v, Q_v mean the powers being consumed by sequent *vth* reducing load-receiver.

A *vth* arc of the main path has been particularly shown in Figure 3.

The notations used in Figure 3 are defined as follows:
l_e — length of an elementary arc (sub-arc), m,
n_{Tv} — number of the MV/LV transformers of the average loads: p_T[kW], q_T[kvar]; it has been assumed that these transformers are equally disposed along the arc length i.e. they are equidistant ($n_{Tv} \geq 0$); in effect, the *vth* arc consists of (n_{Tv}+1) identical sub-arcs,
s_v — sub-arc section, mm²,
P_{sv} — active ***cumulated*** load of *vth* arc, kW; the ***cumulated*** load is the sum of suitable loads of all the load-receivers being supplied from the end node ($N_{f,v}$) of *vth* arc,
Q_{sv} — a. b., but reactive ***cumulated*** load of *vth* arc, kvar,

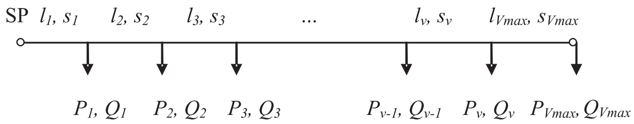


Fig. 2. The example main path (without the branches)

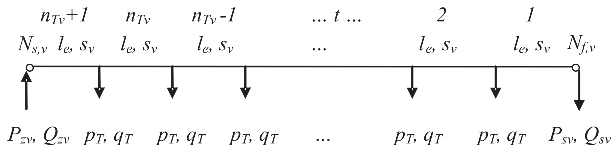


Fig. 3. Particularity of *vth* arc of the main path

P_{zv} — active power supplying (flowing into) the arc $N_{s,v}$–$N_{f,v}$, kW,
Q_{zv} — a. b., but reactive power, kvar.

Each *gth* power line of the power network is characterized by two quantities being assign-ed to the *second category* load-receivers i.e. ***the transformers of average loads***:

p_T — average active load, kW,
q_T — average reactive load, kvar.

The load-receivers are classified — from the point of view of information about their loads — in the following categories:

— *first category* (I) load-receivers — this category concerns the transformers of known peak loads; batteries of parallel power capacitors and local generators are also included,
— *second category* (II) load-receivers — this one concerns exclusively the transformers of unknown loads i.e. such ones which the average loads: p_T , q_T have been assigned to.

The transformers (I) are localized in the power line nodes because the load may be assigned exclusively to the node. The average loads, assigned to the transformers (II), are equidistantly localized in all these arcs (of the power line), in which the number n_T is greater than zero. The average loads have been defined below:

$$p_T = \frac{P_X - \sum P_o}{N_T}, \quad q_T = \frac{Q_X - \sum Q_o - Q_x}{N_T} \qquad (7)$$

where:

N_T — number of the transformers (II) in the power line (of M arcs), equal to:

$$N_T = \sum_{v=1}^{M} n_{Tv} \qquad (8)$$

P_x, Q_x — active and reactive power (respectively), supplying the power line,
ΣP_o — sum of the active loads, assigned to the load-receivers (I),
ΣQ_o — sum of the reactive loads, assigned to the load-receivers (I),
Q_x — charging power (capacitive, negative) of the power line.

If N_T (8) is equal to zero, then average loads (7): p_T, q_T are equal to zero too.

According to the definitions introduced above, the loads being carried over by *tth* sub-arc (t = 1, 2, ..., n_{Tv}+1) are as follows (respectively):

$$P_{vt} = P_{st} + (t-1)\, p_{Tt}, \quad Q_{vt} = Q_{st} + (t-1)\, q_{Tt} \qquad (9)$$

The voltage drop arising in *tth* sub-arc is equal to:

$$\Delta u_{vt} = \frac{1}{U_{vt}}\left(\left(P_{vt}\frac{l_e}{\gamma s_v} + Q_{vt} l_e x_0 \cdot 10^{-3}\right)\right. \qquad (10)$$

where:

U_{vt} — voltage at the end of *tth* sub-arc,
l_e — length of sub-arc, defined as $l_e = \dfrac{l_v}{n_{Tv}+1}$.

The voltage drop in *vth* arc is calculated according to the formula:

$$\Delta u_v = \sum_{t=1}^{n_{Tv}+1} \Delta u_{vt} \qquad (11)$$

Inserting (10) into (11) gives the result:

$$\Delta u_v = l_e\left(\sum_{t=1}^{n_{Tv}+1}\frac{1}{\gamma s_v}\frac{P_{vt}}{U_{vt}} + x_0 \cdot 10^{-3}\sum_{t=1}^{n_{Tv}+1}\frac{Q_{vt}}{U_{vt}}\right);$$

replacing of voltage U_{vt} by an unknown average voltage U_v (and summing) brings on:

$$\Delta u_v = \frac{l_e}{U_v}\left(\frac{1}{\gamma s_v}\left(n_{Tv}+1\right)\left(P_{sv} + \frac{n_{Tv}}{2}p_T\right) + x_0 \cdot 10^{-3}\left(n_{Tv}+1\right)\left(Q_{sv} + \frac{n_{Tv}}{2}q_T\right)\right) \qquad (12)$$

Having inserted length l_e (c.f. (10)) into (12), one obtains its final form:

$$\Delta u_v = \frac{l_v}{U_v}\left(\frac{1}{\gamma s_v}\left(P_{sv} + \frac{n_{Tv}}{2}p_T\right) + x_0 \cdot 10^{-3}\left(Q_{sv} + \frac{n_{Tv}}{2}q_T\right)\right) \qquad (13)$$

Let the equivalent length of $N_{s,v} - N_{f,v}$ arc be signified l_{xv}; at its end node there is the load-receiver P_{zv}, Q_{zv} being supplied with the voltage U_v. The voltage drop arising in the arc l_{xv} is calculated according to the formula:

$$\Delta u_{xv} = \frac{1}{U_v}\left(P_{zv}\frac{l_{xv}}{\gamma s_v} + Q_{zv} l_{xv} x_0 \cdot 10^{-3}\right) \qquad (14)$$

In effect of the equating of formulae (13) and (14), the following result has been obtained:

$$l_{xv} = \frac{l_v\left(\frac{1}{\gamma s_v}\left(P_{sv} + \frac{n_{Tv}}{2}p_T\right) + x_0 \cdot 10^{-3}\left(Q_{sv} + \frac{n_{Tv}}{2}q_T\right)\right)}{\frac{P_{zv}}{\gamma s_v} + Q_{zv} x_0 \cdot 10^{-3}} \qquad (15)$$

where:

$$P_{zv} = P_{sv} + n_{Tv}\, p_T, \quad Q_{zv} = Q_{sv} + n_{Tv}\, q_T$$

Formula (15) allows to substitute each real arc of the main path by the equivalent one of different length l_{xv}; having applied (15) to the sequent arcs, one obtains the equivalent main path (of *gth* power line) which has been demonstrated in Figure 4.

The load P_{zv}, Q_{zv} causes the voltage drop Δu_{xv} arising in the arc l_{xv}:

$$\Delta u_{xv} = \frac{1}{U_{xv}}\left(P_{zv}\frac{l_{xv}}{\gamma s_v} + Q_{zv} l_{xv} x_0 \cdot 10^{-3}\right) \qquad (16)$$

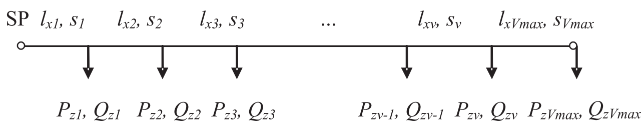


Fig. 4. Substitution of the real arcs by the equivalent ones in the main path

where:

U_{xv} — voltage in the end node of *vth* arc.

On the basis of (16), the total voltage drop in the equivalent main path is equal to:

$$\Delta u_x = \sum_{v=1}^{V_{max}} \left(\frac{1}{\gamma s_v} \frac{P_{zv} l_{xv}}{U_{xv}} + \frac{Q_{zv} l_{xv}}{U_{xv}} x_0 \cdot 10^{-3} \right) \quad (17)$$

The equivalent main path should be now reduced to the single arc of length L_x and section σ (cf. (6)); the course of proceeding is as follows:

a) modification of formula (17) gives the result:

$$\Delta u_x = \frac{1}{U_x} \left(\frac{1}{\gamma} \sum_{v=1}^{V_{max}} \frac{P_{zv} l_{xv}}{s_v} + x_0 \cdot 10^{-3} \sum_{v=1}^{V_{max}} Q_{zv} l_{xv} \right) \quad (18)$$

where:

U_x — average voltage in the main path.

Formula (18) expresses the voltage drop calculated at the assumption $U_{xv} = U_x$;

b) if the main path is reduced to the arc of unknown length L_x — to the end of which the load-receiver, representing the load P_x, Q_x and supplying with the voltage U_x, is connected – then the voltage drop arising in this arc will be calculated according to the formula:

$$\Delta u_x = \frac{1}{U_x} \left(P_X \frac{L_x}{\gamma \sigma} + Q_X L_x x_0 \cdot 10^{-3} \right) \quad (19)$$

where:

P_x, Q_x — active and reactive power (respectively), supplying the power line;

c) on the basis of formulae (18) and (19), the arc length L_x (to be found) is equal to:

$$L_x = \frac{\frac{1}{\gamma} \sum_{v=1}^{V_{max}} \frac{P_{zv} l_{xv}}{s_v} + x_0 \cdot 10^{-3} \sum_{v=1}^{V_{max}} Q_{zv} l_{xv}}{P_X \frac{1}{\gamma \sigma} + Q_X x_0 \cdot 10^{-3}} \quad (20)$$

The index g (reserved for a power line) has been neglected in the above considerations; its return makes the formula (20) get the form:

$$L_{xg} = \frac{\frac{1}{\gamma} \sum_{v=1}^{V_{max\,g}} \frac{P_{zvg} l_{xvg}}{s_{vg}} + x_0 \cdot 10^{-3} \sum_{v=1}^{V_{max\,g}} Q_{zvg} l_{xvg}}{P_{Xg} \frac{1}{\gamma \sigma_g} + Q_{Xg} x_0 \cdot 10^{-3}} \quad (21)$$

Formula (21) allows to substitute each *gth* power line with the radius of length L_{xg}; this radius supplies the load-receiver P_{xg}, Q_{xg} connected to its end. The being considered SP provides with the electrical power G such radii of length L_{xg}, section σ_g, and loads: P_{xg}, Q_{xg}. The average voltage drop Δu_{avg} may be figured out according to the formula:

$$\Delta u_{avg} = \frac{1}{G} \left(\frac{1}{\gamma U} \sum_{g=1}^{G} \frac{P_{Xg} L_{xg}}{\sigma_g} + \frac{x_0 \cdot 10^{-3}}{U} \sum_{g=1}^{G} Q_{Xg} L_{xg} \right) \quad (22)$$

where:

U — assumed reference voltage (the rated voltage, for instance).

These G radii (reduced main paths) should be now substituted with the supply radius (of length L and section σ), carrying over the load P_z, Q_z (cf. Fig. 1) to the load-receiver connected to its end. The voltage drop Δu_{SR} — arising in the supply radius to be found – is equal to:

$$\Delta u_{SR} = \frac{L}{U} \left(\frac{P_Z}{\gamma \sigma} + Q_Z x_0 \cdot 10^{-3} \right) \quad (23)$$

where:

$$P_Z = \sum_{g=1}^{G} P_{Xg}, \quad Q_Z = \sum_{g=1}^{G} Q_{Xg}$$

σ — average section determined as:

$$\sigma = \sum_{g=1}^{G} L_{xg} \sigma_g \cdot \left(\sum_{g=1}^{G} L_{xg} \right)^{-1} \quad (24)$$

After equating of formulae (22) and (23), the length of the supply radius is equal to:

$$L = \frac{\frac{1}{G} \left(\frac{1}{\gamma} \sum_{g=1}^{G} \frac{P_{Xg} L_{xg}}{\sigma_g} + x_0 \cdot 10^{-3} \sum_{g=1}^{G} Q_{Xg} L_{xg} \right)}{\frac{P_Z}{\gamma \sigma} + Q_Z x_0 \cdot 10^{-3}} \quad (25)$$

Length L (25) enables determining of the parameters: R_z, X_z of the supply radius substituting the analyzed power network:

$$R_Z = \frac{L}{\gamma \sigma} \approx R_L, \quad X_Z = L x_0 \cdot 10^{-3} \approx X_L \quad (26)$$

Consequently, quotient m (1) has been finally determined below:

$$m = \gamma \sigma x_0 \cdot 10^{-3} \quad (27)$$

4. VERIFICATION OF THE METHOD

In order to exemplify verification of the presented method of the power network transfiguration, the demonstrative network has been submitted (c.f. Fig. 5), and the necessary calculations have been accomplished. The network consists of four overhead power lines: SP-AA (black), SP-BB (red), SP-CC (green), SP-DD (blue); their conductors are made of aluminum wires, strengthened by steel cores. The conductivity γ amounts 34.8 [m·Ω^{-1}·mm^{-2}], and the specific reactance – 0.4046 [Ω·km^{-1}]. In Figure 5, each arc of each power line has been described by three parameters: length [km], section [mm^2], number of the transformers (II). Under the first arc of each line, there are the powers: active [kW], reactive [kvar], supplying this line.

The results of the calculations have been placed in Table 1. It contains the real voltage drops, figured out for each power line for five different levels of the supplying voltage U_z (U_N = 15.0 kV). It has been assumed that both the supplying powers and the being consumed powers of the power lines are independent of the voltage U_z. The suitable voltage drop has been obtained, after determining the supply radius parameters: L = 2325 m (25), σ = 58.946 mm^2 (24), on the basis of (cf. Fig. 1):

$$\Delta u_{SR} = U_z - U_o$$

where:

$$U_o = \sqrt{\frac{U_z^2 - 2\left(P_Z R_z + Q_Z X_z\right) + \sqrt{\Delta}}{2}}$$

$$\Delta = \left[U_z^2 - 2\left(P_Z R_z + Q_Z X_z\right)\right]^2 -$$

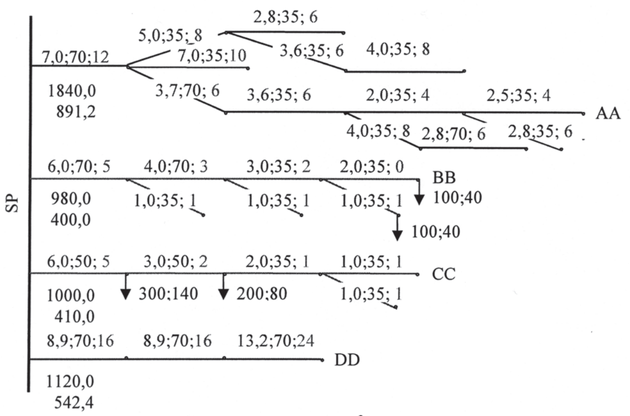
$$-4\left(P_Z^2 + Q_Z^2\right)\left(R_z^2 + X_z^2\right)$$

The relative error $\delta(\Delta u)$, informing on exactness of the method, has been defined below:

$$\delta(\Delta u) = \frac{\Delta u_{avg} - \Delta u_{SR}}{\Delta u_{avg}} \cdot 100\%$$

The last column shows that the relative error $\delta(\Delta u)$ is highly stable; the more the particular power lines of the network differ by their voltage drops, the greater is the relative error value, which means that the supply radius, substituting this network, has been less precisely determined (and *vice versa*).

The method has one remarkable attribute – it allows to determine the substitutive supply radius parameters independently of the supplying voltage level. On the other hand, in spite of its simplicity, the method cannot be too comfortably applied; its application would be much easier if the computer modelling of a power network and the required algorithms were worked out. Out of regard for the less or more outstanding attributes of the method, it has been implemented in the author's computer program *EkoStrop* that is the inheritor of *STROP* [1] and deals with miscellaneous problems of the MV power networks.



l – length [km], *s* – section [mm^2], n_T – number of the transformers (II).

Fig. 5. The MV demonstrative power network — exemplary basis of the method verification

REFERENCE

1. C z e p i e l S.: *Obliczenia optymalizacyjne i inżynierskie dla sieci średniego napięcia. Instrukcja obsługi programu STROP*, Częstochowa, grudzień 1999.
2. H e l l m a n n W., S z c z e r b a Z.: *Regulacja częstotliwości i napięcia w systemie elektroenergetycznym*, Warszawa, WNT 1978.
3. H o r a k J.: *Sieci elektryczne. Sieć rozdzielcza jako zbiór elementów*, Wydawnictwo Politechniki Częstochowskiej, Częstochowa 1991.

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Table 1. Exactness of the method

Voltage level U_z	Real voltage drop in the power lines [kV]				Δu_{avg}	U_o	Δu_{SR}	$\Delta(\Delta u)$
	SP-AA	SP-BB	SP-CC	SP-DD	[kV]	[kV]	[kV]	[%]
15.300	0.799	0.341	0.330	0.687	0.53925	14.778	0.522	3.20
15.450	0.791	0.338	0.327	0.681	0.53425	14.933	0.517	3.23
15.600	0.785	0.334	0.323	0.675	0.52925	15.088	0.512	3.26
15.750	0.777	0.331	0.321	0.668	0.52425	15.244	0.506	3.48
15.900	0.769	0.328	0.318	0.664	0.51975	15.399	0.501	3.61